The All-Purpose Calculus Problem

by

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A particle starts at rest and moves with velocity along a 10 foot ladder, which leans against a trough with a triangular cross-section two feet wide and one foot high. Sand is flowing out of the trough at a constant rate of 2 cubic feet per hour, forming a conical pile in the middle of a sandbox which has been formed by cutting a square of side *x* from each corner of an 8" by 15" piece of cardboard and folding up the sides. An observer watches the particle from a lighthouse one mile offshore, peering through a window shaped like a rectangle surmounted by a semicircle.

A) How fast is the tip of the shadow moving?

B) Find the volume of the solid generated when the trough is rotated about the *y*-axis.

C) Justify your answer.

D) Using the information found in parts A, B, and C, sketch the curve on a pair of coordinate axes.