COMPOSITE LIST OF CONCEPTS, THEOREMS, FORMULAS FOR AP CALCULUS

1. Odd and even functions; symmetry

a) f(x) is even if f(-x) = f(x)
b) f(x) is odd if f(-x) = - f(x)
c) Symmetry:
Even: y-axis symmetry
Odd: origin symmetry

2. Test for continuity

f(x) is continuous at x = a if each of the following conditions are met:

a) $f(a)$ exists
b) $\lim_{x\to a} f(x)$ exists
c) $\lim_{x\to a} f(x) = f(a)$

3. Derivatives as a limit

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

4. First Derivative Test

a) f(x) has an extremum at a if $f'(a) = 0$ or undefined and $f'(x)$ changes sign around a.
b) f(x) increases where $f'(x) > 0$; f(x) decreases where $f'(x) < 0$.

5. Second Derivative Test

a) If $f'(a) = 0$ or undefined, then
1) $f''(a) > 0 \Rightarrow$ min at $x = a$
2) $f''(a) < 0 \Rightarrow$ max at $x = a$

b) f has an inflection point at x = a and $f''(x) = 0$ and $f''(x)$ changes sign around x = a.
c) f is concave up where $f''(x) > 0$; f is concave down where $f''(x) < 0$.

6. Asymptotes

a) Vertical: At x = a if $\lim_{x\to a} f(x) = \pm\infty$
b) Horizontal: At y = b if $\lim_{x\to \pm\infty} f(x) = b$

7. Extreme Value Theorem (Absolute max/min)

For f(x) continuous on [a, b], extrema (absolute max/min) occur among:

a) f(a) or f(b) (endpoints of given interval)
b) where $f'(c) = 0$ or undefined (at critical points)

8. Mean Value Theorem

For f continuous on [a, b] and differentiable on (a, b), there exists c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

9. Rolle's Theorem

For f continuous on [a, b] and differentiable on (a, b) , there exists c in (a, b) such that if $f(a) = f(b) = 0$, then $f'(c) = 0$.

10. Rectilinear Motion

a) $v(t) = x'(t)$; $x(t) = \int v(t)dt$
b) $a(t) = v'(t) = x'(t)$; $v(t) = \int a(t)dt$
c) Particle moves: Right when v(t) > 0
Left when v(t) < 0
d) $speed = |v(t)|$
e) Particle speeds up when a(t) and v(t) are the same sign; Particle slows down when a(t) and v(t) are opposite signs.

11. Equation of a tangent line to a curve

If y = f(x), then the tangent line at $x = a$ is:

$$y - f(a) = f'(a)(x - a)$$

12. Area of a Region of a Plane

For a region bounded by f and g (f >g) over vertical boundaries x = a and x = b:

$$Area = \int_a^b [f(x) - g(x)]dx$$

13. Volume of a Solid of Revolution

a) Circular Disk (Coin) $V = \pi \int_a^b r^2 dr$
b) Circular Ring (Washer) $V = \pi \int_a^b [R^2 - r^2]dr$
c) Cross-Section $V = \int_a^b [\text{Area of cross section}]$

14. Average Value of a Function

$$\text{Average Value } = \frac{1}{b-a}\int_a^b f(x)dx$$