FIVE Theorems You Need to Know !!!!!

1. The Mean Value Theorem (MVT)

The function f(x) must be:
1. **continuous** (a≤x≤b) and
2. **differentiable** (a<x<b)

Then there is at least one number, c, a<c<b such that:

$$f'(c)=\frac{f(b)-f(a)}{b-a}$$

Remember:
Derivative equals slope

Example:

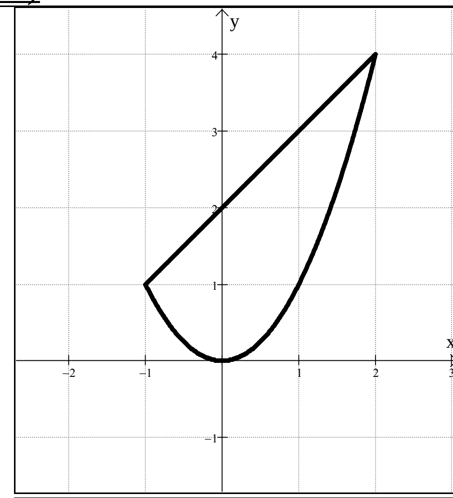
Find c if $f(x)=x^2$ if $-1<c<2$

Solution:

$$f'(c)=2c=\frac{f(2)-f(-1)}{2-(-1)}$$

$$\therefore 2c=\frac{4-1}{3} \quad \Rightarrow \quad \boxed{c=\frac{1}{2}}$$

This means that at $x=\frac{1}{2}$, the derivative of $f(x)$ equals the slope from –1<x<2 !!!!!



2. Rolle's Theorem

The function f(x) must be:
1. **continuous** (a≤x≤b) and
2. **differentiable** (a<x<b) AND
3. $f(a)=f(b)$

Then there is at least one number, c, a<c<b such that:

$$f'(c)=\frac{f(b)-f(a)}{b-a}=0$$

Remember:
Derivative equals slope = 0

Example:

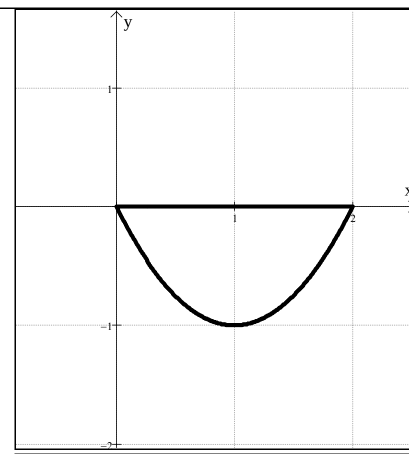
Find c if $f(x)=x^2-2x$ if $0<c<2$

Solution:

$$f(0)=f(2)$$

$$f'(c)=2c-2=0 \quad \Rightarrow \quad \boxed{c=1}$$

This means that at $x=1$, the derivative of $f(x)$ equals 0.



3. The Intermediate Value Theorem (IVT)

The function f(x) must be:
1. **continuous** (a≤x≤b) and
2. m is any number between f(a) and f(b)

Then there is at least one number, c, between a and b such that: f(c)=m

Remember:
If x is "between" min and max x, then y is "between" min and max y.

Example:

Given:

$f(x)=x^2-2x$ if $-1\le c\le 1$

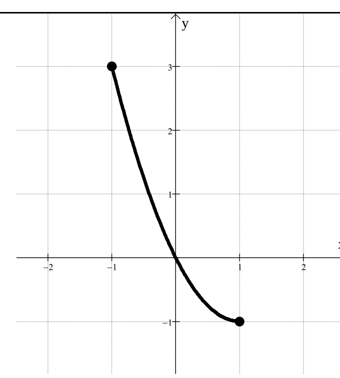
Find the possible values of $f(x)$?

Solution: $f(-1)\ge f(x)\ge f(1)$

So, $3\ge f(x)\ge -1$

This means that $-1\le f(x)\le 3$

Somewhere on the closed interval $-1\le c\le 1$



FIVE Theorems You Need to Know !!!!!

4. Average Value Theorem

We call this one the
"Average Y Value Theorem"

This theorem allows you to calculate the "**average y value**"

Formula = $\frac{1}{b-a}\int_a^b f(x)\,dx$

Remember:
Always use this to find the "Average <u>Y</u> Value"

Example:

Find the <u>average junk</u> if

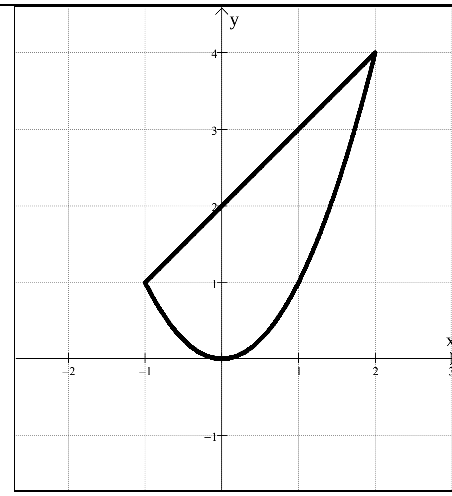
$\text{junk}(x) = x^2$ if $-1 < x < 2$

Solution:

$$AV = \frac{1}{2-(-1)}\int_{-1}^{2} x^2\,dt = \frac{1}{3}\left[\frac{x^3}{3}\right]_{-1}^{2} =$$

$$\frac{1}{3}\left[\frac{8}{3}-\left(\frac{-1}{3}\right)\right] = \frac{1}{3}\left[\frac{9}{3}\right] = \boxed{1}$$

This means that the average junk equals one when $-1 < t < 2$



5. Test for Continuity

A function f, which is defined in some neighborhood of c, is said to be **continuous** at c if :

1) f(c) exists
2) $\lim_{x\to c}$ f(x) exists
3) $\lim_{x\to c}$ f(x) = f(c)

Remember:
If you can graph $f(x)$ without lifting your pencil, then $f(x)$ is continuous

Example:

Is $f(x) = \begin{cases} \frac{x^2-1}{x+1} & x \neq -1 \\ 2 & x = -1 \end{cases}$

CONTINUOUS at $x = -1$?

Solution:

1. $f(x) = -1$
2. $\lim_{x\to -1} f(x) = -2$
3. $f(x) \neq \lim_{x\to -1} f(x)$

This means that $f(x)$ is NOT continuous at x = –1