Sets of Numbers

Imaginary (or Complex) Numbers: Has i (which = $\sqrt{-1}$) in it

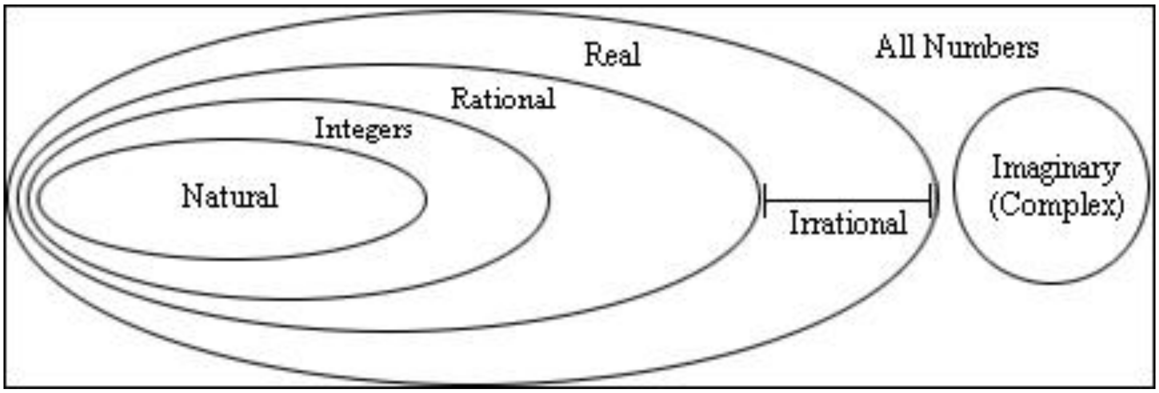
Real Numbers: Everything except imaginary

Irrational Numbers: Non-terminating, non-repeating decimals like π, e, or $\sqrt{2}$

Rational Numbers: Can be written as a fraction $\frac{a}{b}$

Integers: All whole numbers (including zero and negatives)

Natural Numbers: Positive whole numbers



Interval Notation

An interval is a piece of the number line (like all numbers between 2 and 5)

Write it as the two endpoints separated by a comma in parentheses

For a $\leq$ or $\geq$, use square parentheses. For a < or >, use regular parentheses

$2 < x \leq 5 \quad \Leftrightarrow \quad (2, 5]$

$\cup$ = union (combination of two intervals)

$\cap$ = intersection (overlap of two intervals)

Write in interval notation:

$-3 < x \leq 5$ $\qquad$ $x > 5$ $\qquad$ $x < -3 \text{ or } x \geq 2$

Distance Formula

To find the length of a segment using its endpoints

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint Formula

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Find the distance between the points (5,-1) and (-2,7). Then find the midpoint.

Factoring

To solve an equation by factoring, it must be equal to zero!!!

Some common factoring patterns to know:

$u^2 - v^2 = (u+v)(u-v)$

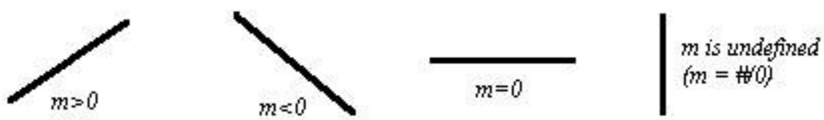
$u^3 + v^3 = (u+v)(u^2 - uv + v^2)$

$u^3 - v^3 = (u-v)(u^2 + uv + v^2)$

Solve for x: $9x^2 - 15x = -6$	Factor and simplify: $27x^3 - y^6$

Slope

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{rise}{run} = \text{"}rate\ of\ change\text{"}$$



Lines

Slope-intercept form: $y = mx + b$

Point-Slope form: $(y - y_1) = m(x - x_1)$

Standard form: $Ax + By = C$ (A, B and C are non-zero whole numbers)

Vertical line: $x = constant$

Horizontal line: $y = constant$

Parallel Lines: $m_1 = m_2$

Perpendicular Lines: $m_1 \cdot m_2 = -1$ $\left(i.e.\quad m_1 = -\frac{1}{m_2} \right)$

Find the equation of the line with a slope of -3 that passes through the point (-2, 5)

Intercepts

To find an intercept of any graph, set all other variables equal to zero and solve for the remaining variable

Find the x-intercept(s) of the graph of $y = x^3 - 4x$

Find the y-intercept

Functions

Function: An equation that relates 2 variables where each input has exactly one output ("vertical line test" says that if a vertical line crosses a graph more than once, then it is *not* a function)

Domain: Any *x* that can be put into the function and give a valid output (remember, the easiest way to find domain is you ask yourself what x can *not* be)

$\mathbb{R}$: All Real numbers = $(-\infty, \infty)$

$\emptyset$: The empty set (nothing)

$\{\mathbb{R} | x \leq 5\}$: "The set of all real numbers *x*, such that *x* is less than or equal to 5"

Range: All the values that *f(x)* takes on.

The easiest way to find range is using a graph. Look for a "floor" or "ceiling" the graph doesn't pass

Find the domain of $f(x) = \sqrt{x^2 - 4}$

Use a calculator to find the range

Find the domain of $g(x) = \frac{1}{|x|-1}$

Find the domain of $h(x) = \frac{x}{\sqrt{9-x^2}}$

Function Notation: x is the input, and $f(x)$ is the output (basically, $f(x)$ is y)

For $f(x) = x^2 + 7$, find $\frac{f(x+h)-f(x)}{h}$. (Note: this is called the difference quotient, and is important)

Restricting Domain

Sometimes you only want to look at a piece of a function. You can do this just by telling the reader what x can be.

For example:

$$f(x) = x^2, \quad x \geq 0$$

Would be the right side of a parabola

Write an equation for the graph of the line <u>segment</u> that connects the points (2,5) and (-7,1).

Piecewise Functions

A function with more than one equation.

Each equation is used over a separate interval (just like restricting domain)

If $f(x) = \begin{cases} 2x-3, & x \leq 0 \\ x^2+1, & x > 0 \end{cases}$

$f(-2) =$ $\quad$ $f(3) =$ $\quad$ $f(0) =$

Absolute Value

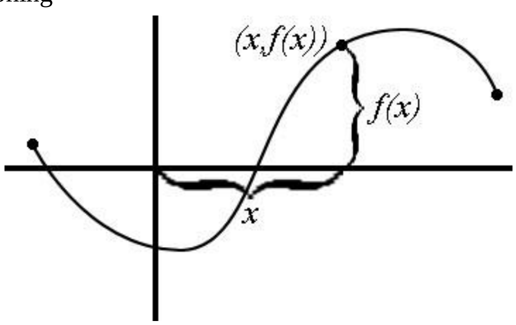
$|x|$ is the <u>distance</u> from zero to a number x on a number line (direction doesn't matter)

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

Solve for x: $|5x - 3| < 23$

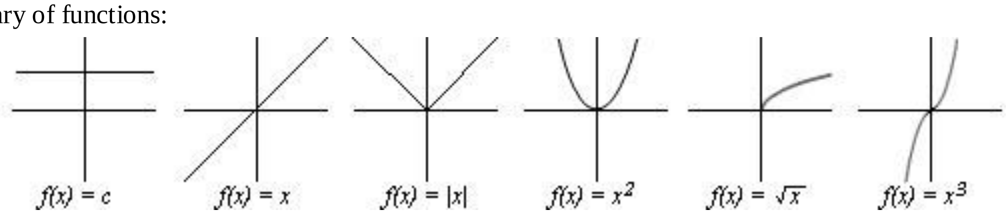
Express $|2x - 3|$ as a piecewise function

Graphing



x is the horizontal component (distance right or left)
$f(x)$ is the vertical component (height) at x

Library of functions:



Transformations

$y = f(x) + C$ shift up if C is positive, down if C is negative

$y = f(x + C)$ shift left if C is positive, right if C is negative

$y = C \cdot f(x)$ stretch ↕ by a factor of C

$y = f(Cx)$ stretch ↔ by a factor of C

$y = -f(x)$ reflect about the x-axis

$y = f(-x)$ reflect about the y-axis

* reflect and stretch before you shift!

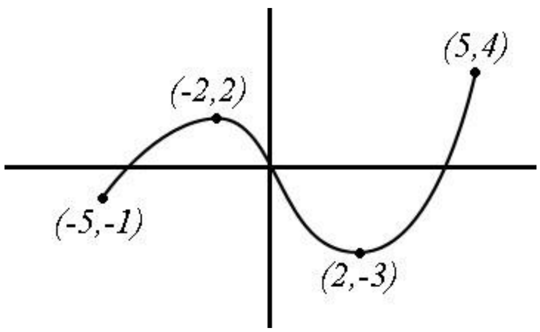
Even Function: $f(-x) = f(x)$ and the graph is symmetric about the y-axis
Odd Function: $f(-x) = -f(x)$ and the graph is symmetric about the origin (180^o rotation)
Note: cos is even, while sin and tan or odd. Reciprocals are the same.
Increasing: If $x_1 > x_2$, then $f(x_1) > f(x_2)$ ("Rising" over a certain interval)
Decreasing: If $x_1 > x_2$, then $f(x_1) < f(x_2)$ ("Falling" over a certain interval)
Constant: For any x_1, x_2, $f(x_1) = f(x_2)$ ("Flat" over a certain interval)

Is each function even, odd or neither?

$f(x) = 3x^2 - 7$

$f(x) = 3(x-2)^3$

Use the graph below to answer the questions.



1. Does the graph represent a function?
2. If so, what is its domain?
3. If so, what is its range?
4. On what interval(s) is the graph increasing?
5. When $x = -2$, what is $f(x)$?
6. For what value(s) of x does $f(x) = -3$?
7. Draw $f(x + 2)$ on the graph above

Compositions:

$(f \circ g)(x) = f(g(x))$

Note: To find the domain of $f(g(x))$ you must also consider the domain of g (the second function)

Find the domain of $(f \circ g)(x)$ if $f(x) = \frac{2}{x-2}$ and $g(x) = 2x - 1$

Find the domain of $(g \circ f)(x)$

Inverses

* To find a functions inverse, switch x and y, and solve for the new y.

Notation: $f^{-1}(x)$

Definition: $f\left(f^{-1}(x)\right) = f^{-1}\left(f(x)\right) = x$

Graphs of inverse functions are reflections of each other in the line $y = x$

Important note:

Domain of $f(x)$ = Range of $f^{-1}(x)$

Range of $f^{-1}(x)$ = Domain of $f(x)$

If $f(x) = x^3 + 4$, find $f^{-1}(x)$

If $f(1) = 5,\ f(3) = 7,\ and\ f(8) = -10$

$f^{-1}(7) =$ $f^{-1}(5) =$ $f^{-1}(-10) =$

Properties of Exponents

$a^0 = 1$ $a^{x+y} = a^x a^y$

$a^{-n} = \frac{1}{a^n}$ $a^{x-y} = \frac{a^x}{a^y}$

$a^{\frac{p}{q}} = \sqrt[q]{a^p}$ $a^{xy} = (a^x)^y$

$(ab)^x = a^x b^x$

True or false: $\left(\frac{1}{2}\right)^x = 2^{-x}$

Evaluate: $25^{-\frac{1}{2}}$

Logarithms

$log_5 x$ is read "log base 5 of x" and means "5 to what power equals x?"

b can be any positive number except 1

You can not take the log of a negative number or zero (domain is $x > 0$)

Logs and exponentials are inverses!

Properties of Logarithms (For any $b > 0$)

1. $log_b 1 = 0$
2. $log_b b = 1$
3. $log_b b^x = x$
4. $b^{(log_b x)} = x$
5. $log_b(xy) = log_b x + log_b y$
6. $log_b\left(\frac{x}{y}\right) = log_b x - log_b y$
7. $log_b(x^c) = c\,log_b x$
8. $log_n m = \frac{log_b m}{log_b n}$ $for\ any\ b$

Converting logs to exponentials

Method 1 – Definition of inverses

Just switch x and y

Method 2 – Canceling out functions

Use property 3 or 4 above

Convert: $log_5 7x = y - 3$ $e^{3x+1} = 20$

Find the domain of:

$log_5(x^2 - 4)$

Solving Log/Exp Equations

Isolate the log/exp function

Switch forms using one of the methods above

Solve for x

Check your answers (extraneous solutions for log solutions)

$5e^x - 3 = 12$	$\ln x - 5 = 2$
$log_b\sqrt{3} = \frac{1}{4}$	$log_2(4x + 10) - log_2(x + 1) = 3$

$e^x - 2e^{-x} = 1$	$-x^2e^{-x} + 2xe^{-x} = 0$
$4^x - 2^x - 2 = 0$	$\frac{30}{2+e^{2x}} = 2$

Trigonometry

Know the unit circle!!!! I'm not going to re-re-re-re-teach it.

Know radian angles ($2\pi = 360^o$)

Know Identities: Reciprocal, Pythagorean, Even/Odd

Evaluate:

$\sin\frac{7\pi}{6}$	$\cos\left(-\frac{\pi}{3}\right)$
$\sin^{-1}\left(\sin\left(-\frac{\pi}{3}\right)\right)$	$\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$
$\cos^2\frac{\pi}{16} + \sin^2\frac{\pi}{16}$	

Solving Trig Equations

Isolate the trig function(s) first (often done by factoring)

Solve using inverse trig (unit circle)

$2\sin x + 1 = 0 \ \ on \ \ [0, 2\pi)$	$2\sin x + \sin^2 x = 0 \ \ on \ \ [0, 2\pi)$
$\cos x + 2\cos^2 x = 1 \ \ on \ \ [0, 2\pi)$	$\cos x + 1 = \sin x \ \ on \ \ [0, 2\pi)$
$2\cos(2x) = 1 \ \ on \ \ [0, 2\pi)$	$3\tan^2 x - 1 = 0 \ \ on \ \ (0, \pi)$