

CALCULUS AB
SECTION I, Part A
Time—55 minutes
Number of questions—28

Name _____
Periods _____

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

In this exam:

- (1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.
- (2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix “arc” (e.g., $\sin^{-1}x = \arcsin x$).

1. $\lim_{x \rightarrow \infty} \frac{(2x-1)(3-x)}{(x-1)(x+3)}$ is

- (A) -3 (B) -2 (C) 2 (D) 3 (E) nonexistent

2. $\int \frac{1}{x^2} dx =$

- (A) $\ln x^2 + C$ (B) $-\ln x^2 + C$ (C) $x^{-1} + C$ (D) $-x^{-1} + C$ (E) $-2x^{-3} + C$

3. If $f(x) = (x-1)(x^2+2)^3$, then $f'(x) =$

- (A) $6x(x^2+2)^2$
(B) $6x(x-1)(x^2+2)^2$ (D) $(x^2+2)^2(7x^2-6x+2)$
(C) $(x^2+2)^2(x^2+3x-1)$ (E) $-3(x-1)(x^2+2)^2$

4. $\int (\sin(2x) + \cos(2x)) dx =$

- (A) $\frac{1}{2}\cos(2x) + \frac{1}{2}\sin(2x) + C$
(B) $-\frac{1}{2}\cos(2x) + \frac{1}{2}\sin(2x) + C$ (D) $2\cos(2x) - 2\sin(2x) + C$
(C) $2\cos(2x) + 2\sin(2x) + C$ (E) $-2\cos(2x) + 2\sin(2x) + C$

5. $\lim_{x \rightarrow 0} \frac{5x^4 + 8x^2}{3x^4 - 16x^2}$ is

- (A) $-\frac{1}{2}$ (B) 0 (C) 1 (D) $\frac{5}{3}$ (E) nonexistent

$$f(x) = \begin{cases} \frac{x^2 - 4}{x - 2} & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$$

6. Let f be the function defined above. Which of the following statements about f are true?

- I. f has a limit at $x = 2$.
 II. f is continuous at $x = 2$.
 III. f is differentiable at $x = 2$.

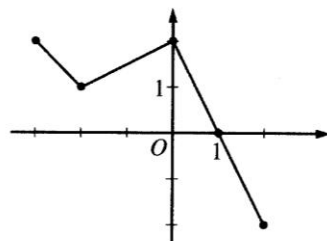
- (A) I only (D) I and II only
 (B) II only (E) I, II, and III
 (C) III only

7. A particle moves along the x -axis with velocity given by $v(t) = 3t^2 + 6t$ for time $t \geq 0$. If the particle is at position $x = 2$ at time $t = 0$, what is the position of the particle at time $t = 1$?

- (A) 4 (B) 6 (C) 9 (D) 11 (E) 12

8. If $f(x) = \cos(3x)$, then $f'\left(\frac{\pi}{9}\right) =$

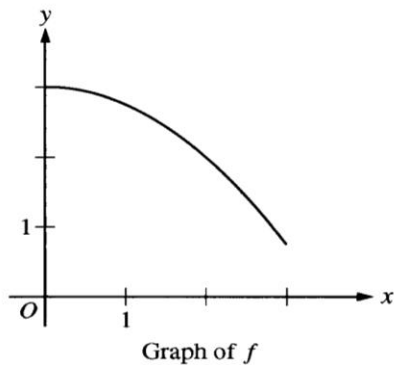
- (A) $\frac{3\sqrt{3}}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $-\frac{\sqrt{3}}{2}$ (D) $-\frac{3}{2}$ (E) $-\frac{3\sqrt{3}}{2}$



Graph of f

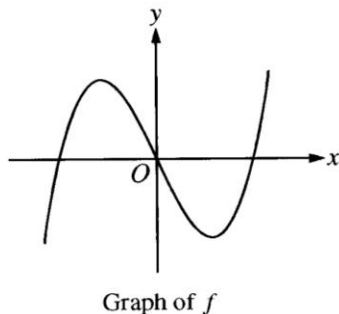
9. The graph of the piecewise linear function f is shown in the figure above. If $g(x) = \int_{-2}^x f(t) dt$, which of the following values is greatest?

- (A) $g(-3)$ (B) $g(-2)$ (C) $g(0)$ (D) $g(1)$ (E) $g(2)$

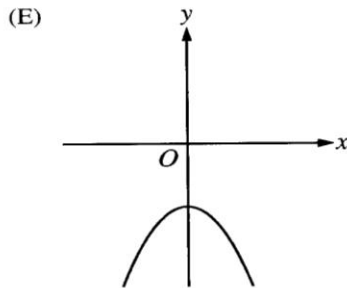
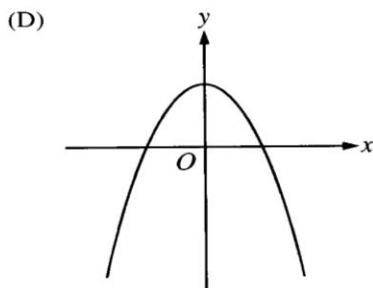
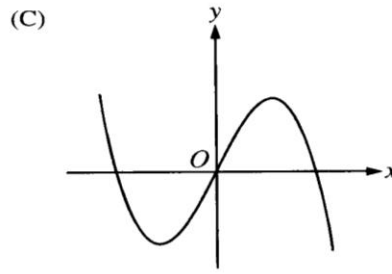
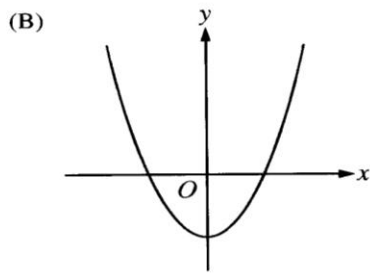
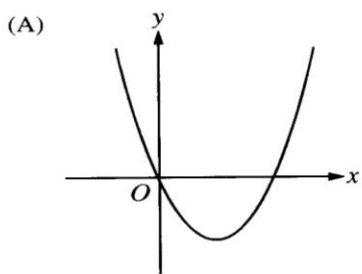


10. The graph of the function f is shown above for $0 \leq x \leq 3$. Of the following, which has the least value?

- (A) $\int_1^3 f(x) dx$
- (B) Left Riemann sum approximation of $\int_1^3 f(x) dx$ with 4 subintervals of equal length
- (C) Right Riemann sum approximation of $\int_1^3 f(x) dx$ with 4 subintervals of equal length
- (D) Midpoint Riemann sum approximation of $\int_1^3 f(x) dx$ with 4 subintervals of equal length
- (E) Trapezoidal sum approximation of $\int_1^3 f(x) dx$ with 4 subintervals of equal length



11. The graph of a function f is shown above. Which of the following could be the graph of f' , the derivative of f ?



12. If $f(x) = e^{(2/x)}$, then $f'(x) =$

- (A) $2e^{(2/x)} \ln x$ (B) $e^{(2/x)}$ (C) $e^{(-2/x^2)}$ (D) $-\frac{2}{x^2}e^{(2/x)}$ (E) $-2x^2e^{(2/x)}$

13. If $f(x) = x^2 + 2x$, then $\frac{d}{dx}(f(\ln x)) =$

- (A) $\frac{2 \ln x + 2}{x}$ (B) $2x \ln x + 2x$ (C) $2 \ln x + 2$ (D) $2 \ln x + \frac{2}{x}$ (E) $\frac{2x + 2}{x}$

x	0	1	2	3
$f''(x)$	5	0	-7	4

14. The polynomial function f has selected values of its second derivative f'' given in the table above.

Which of the following statements must be true?

- (A) f is increasing on the interval $(0, 2)$.
(B) f is decreasing on the interval $(0, 2)$.
(C) f has a local maximum at $x = 1$.
(D) The graph of f has a point of inflection at $x = 1$.
(E) The graph of f changes concavity in the interval $(0, 2)$.

15. $\int \frac{x}{x^2 - 4} dx =$

- (A) $\frac{-1}{4(x^2 - 4)^2} + C$ (D) $2 \ln|x^2 - 4| + C$
(B) $\frac{1}{2(x^2 - 4)} + C$
(C) $\frac{1}{2} \ln|x^2 - 4| + C$ (E) $\frac{1}{2} \arctan\left(\frac{x}{2}\right) + C$

16. If $\sin(xy) = x$, then $\frac{dy}{dx} =$

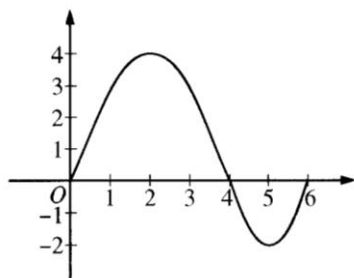
(A) $\frac{1}{\cos(xy)}$

(B) $\frac{1}{x \cos(xy)}$

(C) $\frac{1 - \cos(xy)}{\cos(xy)}$

(D) $\frac{1 - y \cos(xy)}{x \cos(xy)}$

(E) $\frac{y(1 - \cos(xy))}{x}$



Graph of f

17. The graph of the function f shown above has horizontal tangents at $x = 2$ and $x = 5$. Let g be the function defined by $g(x) = \int_0^x f(t) dt$. For what values of x does the graph of g have a point of inflection?

(A) 2 only

(B) 4 only

(C) 2 and 5 only

(D) 2, 4, and 5

(E) 0, 4, and 6

18. In the xy -plane, the line $x + y = k$, where k is a constant, is tangent to the graph of $y = x^2 + 3x + 1$. What is the value of k ?

(A) -3

(B) -2

(C) -1

(D) 0

(E) 1

19. What are all horizontal asymptotes of the graph of $y = \frac{5 + 2^x}{1 - 2^x}$ in the xy -plane?

(A) $y = -1$ only

(B) $y = 0$ only

(D) $y = -1$ and $y = 0$

(C) $y = 5$ only

(E) $y = -1$ and $y = 5$

20. Let f be a function with a second derivative given by $f''(x) = x^2(x - 3)(x - 6)$. What are the x -coordinates of the points of inflection of the graph of f ?

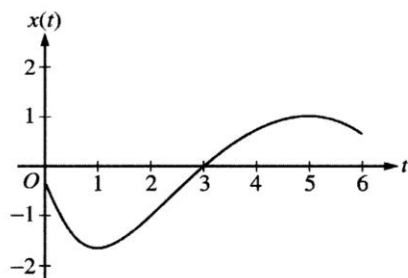
(A) 0 only

(B) 3 only

(C) 0 and 6 only

(D) 3 and 6 only

(E) 0, 3, and 6



21. A particle moves along a straight line. The graph of the particle's position $x(t)$ at time t is shown above for $0 < t < 6$. The graph has horizontal tangents at $t = 1$ and $t = 5$ and a point of inflection at $t = 2$. For what values of t is the velocity of the particle increasing?

- (A) $0 < t < 2$
 (B) $1 < t < 5$ (D) $3 < t < 5$ only
 (C) $2 < t < 6$ (E) $1 < t < 2$ and $5 < t < 6$

22. A rumor spreads among a population of N people at a rate proportional to the product of the number of people who have heard the rumor and the number of people who have not heard the rumor. If p denotes the number of people who have heard the rumor, which of the following differential equations could be used to model this situation with respect to time t , where k is a positive constant?

- (A) $\frac{dp}{dt} = kp$
 (B) $\frac{dp}{dt} = kp(N - p)$ (D) $\frac{dp}{dt} = kt(N - t)$
 (C) $\frac{dp}{dt} = kp(p - N)$ (E) $\frac{dp}{dt} = kt(t - N)$

23. Which of the following is the solution to the differential equation $\frac{dy}{dx} = \frac{x^2}{y}$ with the initial condition $y(3) = -2$?

- (A) $y = 2e^{-9+x^3/3}$
 (B) $y = -2e^{-9+x^3/3}$ (D) $y = \sqrt{\frac{2x^3}{3} - 14}$
 (C) $y = \sqrt{\frac{2x^3}{3}}$ (E) $y = -\sqrt{\frac{2x^3}{3} - 14}$

24. The function f is twice differentiable with $f(2) = 1$, $f'(2) = 4$, and $f''(2) = 3$. What is the value of the approximation of $f(1.9)$ using the line tangent to the graph of f at $x = 2$?

- (A) 0.4 (B) 0.6 (C) 0.7 (D) 1.3 (E) 1.4

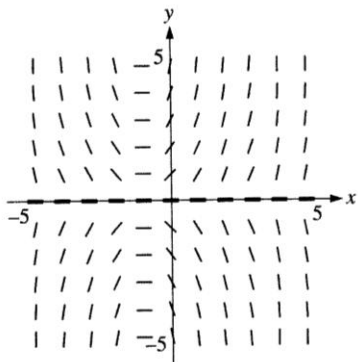
$$f(x) = \begin{cases} cx + d & \text{for } x \leq 2 \\ x^2 - cx & \text{for } x > 2 \end{cases}$$

25. Let f be the function defined above, where c and d are constants. If f is differentiable at $x = 2$, what is the value of $c + d$?

- (A) -4 (B) -2 (C) 0 (D) 2 (E) 4

26. What is the slope of the line tangent to the curve $y = \arctan(4x)$ at the point at which $x = \frac{1}{4}$?

- (A) 2 (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{1}{2}$ (E) -2



27. Shown above is a slope field for which of the following differential equations?

- (A) $\frac{dy}{dx} = xy$
 (B) $\frac{dy}{dx} = xy - y$ (D) $\frac{dy}{dx} = xy + x$
 (C) $\frac{dy}{dx} = xy + y$ (E) $\frac{dy}{dx} = (x + 1)^3$

28. Let f be a differentiable function such that $f(3) = 15$, $f(6) = 3$, $f'(3) = -8$, and $f'(6) = -2$. The function g is differentiable and $g(x) = f^{-1}(x)$ for all x . What is the value of $g'(3)$?

- (A) $-\frac{1}{2}$ (B) $-\frac{1}{8}$ (C) $\frac{1}{6}$ (D) $\frac{1}{3}$
 (E) The value of $g'(3)$ cannot be determined from the information given.

Do not go on to the next question until you are told to do so. You will not be able to return to this part of the test.

CALCULUS AB

SECTION I, Part B

Time—50 minutes

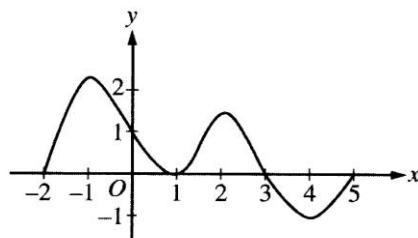
Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:

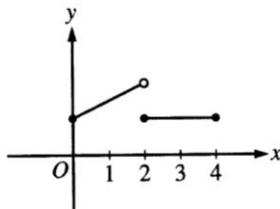
- (1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
- (2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.
- (3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix “arc” (e.g., $\sin^{-1} x = \arcsin x$).



Graph of f'

29. The graph of f' , the derivative of f , is shown above for $-2 \leq x \leq 5$. On what intervals is f increasing?

- (A) $[-2, 1]$ only
(B) $[-2, 3]$ (D) $[0, 1.5]$ and $[3, 5]$
(C) $[3, 5]$ only (E) $[-2, -1]$, $[1, 2]$, and $[4, 5]$



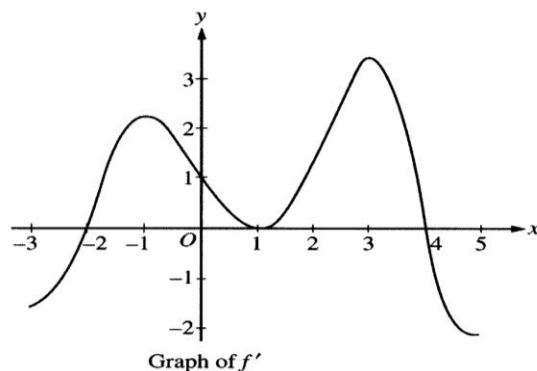
Graph of f

30. The figure above shows the graph of a function f with domain $0 \leq x \leq 4$. Which of the following statements are true?

- I. $\lim_{x \rightarrow 2^-} f(x)$ exists.
II. $\lim_{x \rightarrow 2^+} f(x)$ exists.
III. $\lim_{x \rightarrow 2} f(x)$ exists.
- (A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

31. The first derivative of the function f is defined by $f'(x) = \sin(x^3 - x)$ for $0 \leq x \leq 2$. On what intervals is f increasing?
- (A) $1 \leq x \leq 1.445$ only
 (B) $1 \leq x \leq 1.691$ (D) $0.577 \leq x \leq 1.445$ and $1.875 \leq x \leq 2$
 (C) $1.445 \leq x \leq 1.875$ (E) $0 \leq x \leq 1$ and $1.691 \leq x \leq 2$
32. If $\int_{-5}^2 f(x) dx = -17$ and $\int_5^2 f(x) dx = -4$, what is the value of $\int_{-5}^5 f(x) dx$?
- (A) -21 (B) -13 (C) 0 (D) 13 (E) 21
33. The derivative of the function f is given by $f'(x) = x^2 \cos(x^2)$. How many points of inflection does the graph of f have on the open interval $(-2, 2)$?
- (A) One (B) Two (C) Three (D) Four (E) Five
34. If $G(x)$ is an antiderivative for $f(x)$ and $G(2) = -7$, then $G(4) =$
- (A) $f'(4)$
 (B) $-7 + f'(4)$ (D) $\int_2^4 (-7 + f(t)) dt$
 (C) $\int_2^4 f(t) dt$ (E) $-7 + \int_2^4 f(t) dt$
35. A particle moves along a straight line with velocity given by $v(t) = 7 - (1.01)^{-t^2}$ at time $t \geq 0$. What is the acceleration of the particle at time $t = 3$?
- (A) -0.914 (B) 0.055 (C) 5.486 (D) 6.086 (E) 18.087

36. What is the area enclosed by the curves $y = x^3 - 8x^2 + 18x - 5$ and $y = x + 5$?
- (A) 10.667 (B) 11.833 (C) 14.583 (D) 21.333 (E) 32



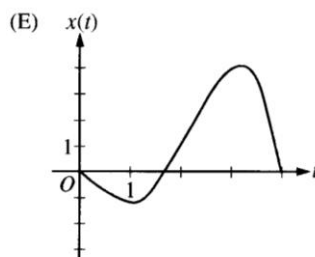
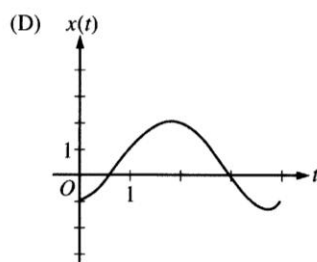
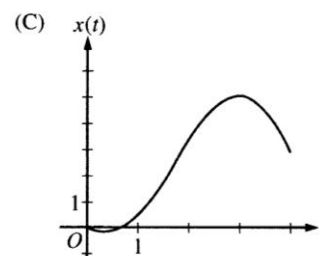
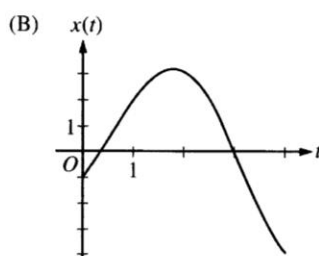
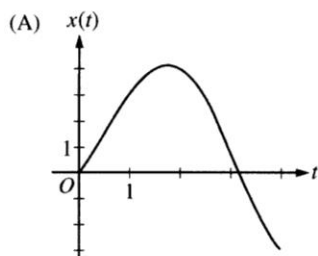
37. The graph of the derivative of a function f is shown in the figure above. The graph has horizontal tangent lines at $x = -1$, $x = 1$, and $x = 3$. At which of the following values of x does f have a relative maximum?
- (A) -2 only (B) 1 only (C) 4 only (D) -1 and 3 only (E) -2 , 1 , and 4

x	-4	-3	-2	-1
$f(x)$	0.75	-1.5	-2.25	-1.5
$f'(x)$	-3	-1.5	0	1.5

38. The table above gives values of a function f and its derivative at selected values of x . If f' is continuous on the interval $[-4, -1]$, what is the value of $\int_{-4}^{-1} f'(x) dx$?
- (A) -4.5 (B) -2.25 (C) 0 (D) 2.25 (E) 4.5

t	0	1	2	3	4
$v(t)$	-1	2	3	0	-4

39. The table gives selected values of the velocity, $v(t)$, of a particle moving along the x -axis. At time $t = 0$, the particle is at the origin. Which of the following could be the graph of the position, $x(t)$, of the particle for $0 \leq t \leq 4$?



40. An object traveling in a straight line has position $x(t)$ at time t . If the initial position is $x(0) = 2$ and the velocity of the object is $v(t) = \sqrt[3]{1+t^2}$, what is the position of the object at time $t = 3$?

(A) 0.431 (B) 2.154 (C) 4.512 (D) 6.512 (E) 17.408

41. The radius of a sphere is decreasing at a rate of 2 centimeters per second. At the instant when the radius of the sphere is 3 centimeters, what is the rate of change, in square centimeters per second, of the surface area of the sphere? (The surface area S of a sphere with radius r is $S = 4\pi r^2$.)

(A) -108π (B) -72π (C) -48π (D) -24π (E) -16π

42. The function f is continuous for $-2 \leq x \leq 2$ and $f(-2) = f(2) = 0$. If there is no c , where $-2 < c < 2$, for which $f'(c) = 0$, which of the following statements must be true?

(A) For $-2 < k < 2$, $f'(k) > 0$.
 (B) For $-2 < k < 2$, $f'(k) < 0$.
 (C) For $-2 < k < 2$, $f'(k)$ exists.
 (D) For $-2 < k < 2$, $f'(k)$ exists, but f' is not continuous.
 (E) For some k , where $-2 < k < 2$, $f'(k)$ does not exist.

43. The function f is continuous on the closed interval $[2, 4]$ and twice differentiable on the open interval $(2, 4)$. If $f'(3) = 2$ and $f''(x) < 0$ on the open interval $(2, 4)$, which of the following could be a table of values for f ?

(A)

x	$f(x)$
2	2.5
3	5
4	6.5

(B)

x	$f(x)$
2	2.5
3	5
4	7

(C)

x	$f(x)$
2	3
3	5
4	6.5

(D)

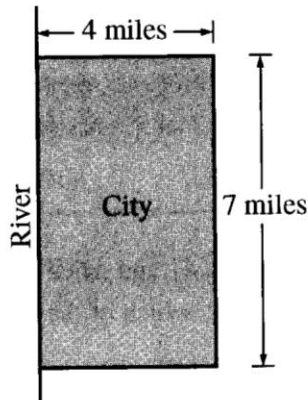
x	$f(x)$
2	3
3	5
4	7

(E)

x	$f(x)$
2	3.5
3	5
4	7.5

44. What is the average value of $y = \frac{\cos x}{x^2 + x + 2}$ on the closed interval $[-1, 3]$?

- (A) -0.085 (B) 0.090 (C) 0.183 (D) 0.244 (E) 0.732



45. A city located beside a river has a rectangular boundary as shown in the figure above. The population density of the city at any point along a strip x miles from the river's edge is $f(x)$ persons per square mile. Which of the following expressions gives the population of the city?

(A) $\int_0^4 f(x) dx$

(B) $7 \int_0^4 f(x) dx$

(D) $\int_0^7 f(x) dx$

(C) $28 \int_0^4 f(x) dx$

(E) $4 \int_0^7 f(x) dx$

AP[®] Calculus AB Exam

SECTION II: Free-Response Questions

At a Glance

Total Time
1 hour, 30 minutes

Number of Questions
6

Percent of Total Grade
50%

Writing Instrument
Either pencil or pen with black or dark blue ink

Weight
The questions are weighted equally, but the parts of a question are not necessarily given equal weight.

Part A

Number of Questions
3

Time
45 minutes

Electronic Device
Graphing calculator required

Percent of Section II Score
50%

Part B

Number of Questions
3

Time
45 minutes

Electronic Device
None allowed

Percent of Section II Score
50%

Instructions

The questions for Part A are printed in the green insert and the questions for Part B are printed in the blue insert. You may use the inserts to organize your answers and for scratch work, but you must write your answers in the pink Section II booklet. No credit will be given for work written in the inserts. Write your solution to each part of each question in the space provided for that part in the Section II booklet. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be graded.

Manage your time carefully. During the timed portion for Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During the timed portion for Part B, you may keep the green insert and continue to work on the questions in Part A without the use of a calculator.

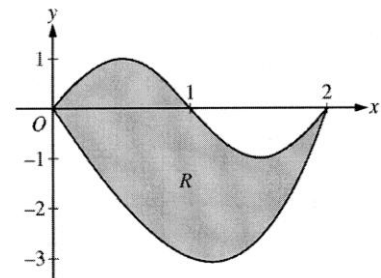
For each part of Section II, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

- Show all of your work. Clearly label any functions, graphs, tables, or other objects that you use. Your work will be graded on the correctness and completeness of your methods as well as your answers. Answers without supporting work may not receive credit. Justifications require that you give mathematical (noncalculator) reasons.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2 dx$ may not be written as $\text{fnInt}(X^2, X, 1, 5)$.
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be graded on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

CALCULUS AB SECTION II, Part A

Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let R be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$, as shown in the figure above.
 - (a) Find the area of R .
 - (b) The horizontal line $y = -2$ splits the region R into two parts. Write, but do not evaluate, an integral expression for the area of the part of R that is below this horizontal line.
 - (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x -axis is a square. Find the volume of this solid.
 - (d) The region R models the surface of a small pond. At all points in R at a distance x from the y -axis, the depth of the water is given by $h(x) = 3 - x$. Find the volume of water in the pond.

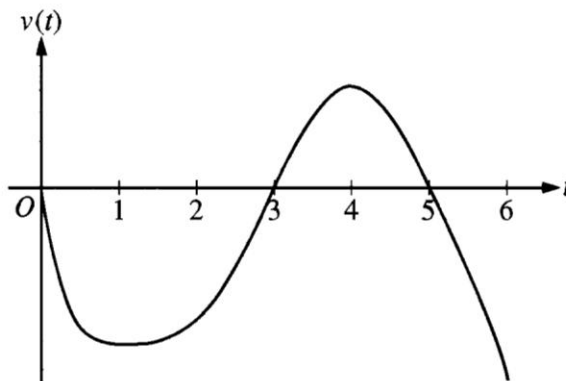
t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

2. Concert tickets went on sale at noon ($t = 0$) and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \leq t \leq 9$. Values of $L(t)$ at various times t are shown in the table above.
- Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. ($t = 5.5$). Show the computations that lead to your answer. Indicate units of measure.
 - Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.
 - For $0 \leq t \leq 9$, what is the fewest number of times at which $L'(t)$ must equal 0? Give a reason for your answer.
 - The rate at which tickets were sold for $0 \leq t \leq 9$ is modeled by $r(t) = 550te^{-t/2}$ tickets per hour. Based on the model, how many tickets were sold by 3 P.M. ($t = 3$), to the nearest whole number?
3. Oil is leaking from a pipeline on the surface of a lake and forms an oil slick whose volume increases at a constant rate of 2000 cubic centimeters per minute. The oil slick takes the form of a right circular cylinder with both its radius and height changing with time. (Note: The volume V of a right circular cylinder with radius r and height h is given by $V = \pi r^2 h$.)
- At the instant when the radius of the oil slick is 100 centimeters and the height is 0.5 centimeter, the radius is increasing at the rate of 2.5 centimeters per minute. At this instant, what is the rate of change of the height of the oil slick with respect to time, in centimeters per minute?
 - A recovery device arrives on the scene and begins removing oil. The rate at which oil is removed is $R(t) = 400\sqrt{t}$ cubic centimeters per minute, where t is the time in minutes since the device began working. Oil continues to leak at the rate of 2000 cubic centimeters per minute. Find the time t when the oil slick reaches its maximum volume. Justify your answer.
 - By the time the recovery device began removing oil, 60,000 cubic centimeters of oil had already leaked. Write, but do not evaluate, an expression involving an integral that gives the volume of oil at the time found in part (b).

Do not go on to the next question until you are told to do so. You will be allowed to continue working on these three problems. However, you will not be allowed to use your calculator.

CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



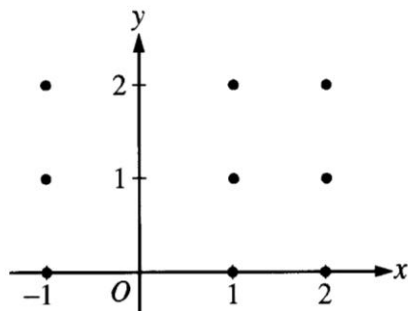
Graph of v

4. A particle moves along the x -axis so that its velocity at time t , for $0 \leq t \leq 6$, is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0$, $t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$. The areas of the regions bounded by the t -axis and the graph of v on the intervals $[0, 3]$, $[3, 5]$, and $[5, 6]$ are 8, 3, and 2, respectively. At time $t = 0$, the particle is at $x = -2$.
- (a) For $0 \leq t \leq 6$, find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.
- (b) For how many values of t , where $0 \leq t \leq 6$, is the particle at $x = -8$? Explain your reasoning.
- (c) On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.
- (d) During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

5. Consider the differential equation $\frac{dy}{dx} = \frac{y-1}{x^2}$, where $x \neq 0$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

(Note: Use the axes provided in the exam booklet.)



(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(2) = 0$.

(c) For the particular solution $y = f(x)$ described in part (b), find $\lim_{x \rightarrow \infty} f(x)$.

6. Let f be the function given by $f(x) = \frac{\ln x}{x}$ for all $x > 0$. The derivative of f is given by $f'(x) = \frac{1 - \ln x}{x^2}$.

(a) Write an equation for the line tangent to the graph of f at $x = e^2$.

(b) Find the x -coordinate of the critical point of f . Determine whether this point is a relative minimum, a relative maximum, or neither for the function f . Justify your answer.

(c) The graph of the function f has exactly one point of inflection. Find the x -coordinate of this point.

(d) Find $\lim_{x \rightarrow 0^+} f(x)$.