CALCULUS BC
SECTION I, Part A
Time—55 minutes
Number of questions—28

Name ____________________

Periods________

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

1. If $f(x) = \dfrac{x^2 + 3x + 2}{x + 3}$, then $f'(x) =$

(A) $2x + 3$

(B) $\dfrac{-x^2 - 6x - 7}{(x+3)^2}$

(C) $\dfrac{x^2 + 6x + 7}{(x+3)^2}$

(D) $\dfrac{x^2 + 12x + 11}{(x+3)^2}$

(E) $\dfrac{3x^2 + 12x + 11}{(x+3)^2}$

2. $\displaystyle\int 5x\left(\sqrt{x} - x^2\right) dx =$

(A) $\dfrac{15\sqrt{x}}{2} - 15x^2 + C$

(B) $5x - \dfrac{5x^4}{4} + C$

(C) $2x^{5/2} - \dfrac{5x^4}{4} + C$

(D) $\dfrac{25x^{5/2}}{2} - \dfrac{5x^4}{4} + C$

(E) $\dfrac{5x^{7/2}}{3} - \dfrac{5x^6}{6} + C$

3. What is the value of $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{5^n}$?

(A) $-\dfrac{15}{8}$ (B) $-\dfrac{9}{8}$ (C) $-\dfrac{3}{8}$ (D) $\dfrac{9}{8}$ (E) $\dfrac{15}{8}$

4. Which of the following is an equation of the line tangent to the graph of $x^2 - 3xy = 10$ at the point $(1, -3)$?

(A) $y + 3 = -11(x - 1)$

(B) $y + 3 = -\frac{7}{3}(x - 1)$

(C) $y + 3 = \frac{1}{3}(x - 1)$

(D) $y + 3 = \frac{7}{3}(x - 1)$

(E) $y + 3 = \frac{11}{3}(x - 1)$

5. If $y = \frac{1}{2}x^{4/5} - \frac{3}{x^5}$, then $\frac{dy}{dx} =$

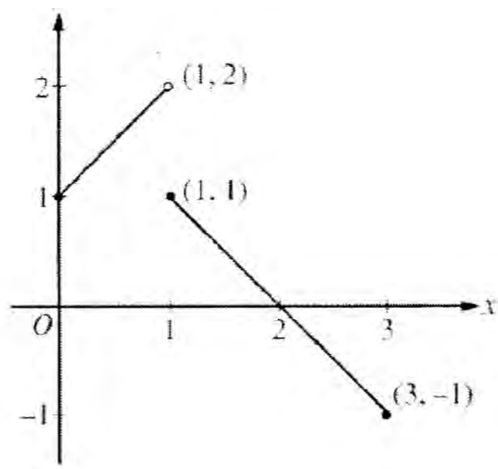
(A) $\frac{2}{5x^{1/5}} + \frac{15}{x^6}$

(B) $\frac{2}{5x^{1/5}} + \frac{15}{x^4}$

(C) $\frac{2}{5x^{1/5}} - \frac{3}{5x^4}$

(D) $\frac{2x^{1/5}}{5} + \frac{15}{x^6}$

(E) $\frac{2x^{1/5}}{5} - \frac{3}{5x^4}$



Graph of f

6. The graph of the function f consists of two line segments, as shown in the figure above. The value of $\int_0^3 |f(x)|\,dx$ is

(A) $-\frac{3}{2}$ (B) $\frac{1}{2}$ (C) $\frac{3}{2}$ (D) $\frac{5}{2}$ (E) nonexistent

7. A population y changes at a rate modeled by the differential equation $\frac{dy}{dt} = 0.2y(1000 - y)$, where t is measured in years. What are all values of y for which the population is increasing at a decreasing rate?

(A) 500 only

(B) $0 < y < 500$ only

(C) $500 < y < 1000$ only

(D) $0 < y < 1000$

(E) $y > 1000$

8. Which of the following gives the length of the path described by the parametric equations $x(t) = 2 + 3t$ and $y(t) = 1 + t^2$ from $t = 0$ to $t = 1$?

(A) $\int_0^1 \sqrt{1 + \frac{4t^2}{9}}\, dt$

(B) $\int_0^1 \sqrt{1 + 4t^2}\, dt$

(C) $\int_0^1 \sqrt{3 + 3t + t^2}\, dt$

(D) $\int_0^1 \sqrt{9 + 4t^2}\, dt$

(E) $\int_0^1 \sqrt{(2 + 3t)^2 + \left(1 + t^2\right)^2}\, dt$

9. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = 2x + y$ with initial condition $f(1) = 0$. What is the approximation for $f(2)$ obtained by using Euler's method with two steps of equal length, starting at $x = 1$?

(A) 0 (B) 1 (C) 2.75 (D) 3 (E) 6

10. If $\int_0^k \frac{x}{x^2 + 4}\, dx = \frac{1}{2}\ln 4$, where $k > 0$, then $k =$

(A) 0 (B) $\sqrt{2}$ (C) 2 (D) $\sqrt{12}$ (E) $\frac{1}{2}\tan\left(\ln \sqrt{2}\right)$

11. The third-degree Taylor polynomial for a function f about $x = 4$ is $\frac{(x-4)^3}{512} - \frac{(x-4)^2}{64} + \frac{(x-4)}{4} + 2$. What is the value of $f'''(4)$?

(A) $-\frac{1}{64}$ (B) $-\frac{1}{32}$ (C) $\frac{1}{512}$ (D) $\frac{3}{256}$ (E) $\frac{81}{256}$

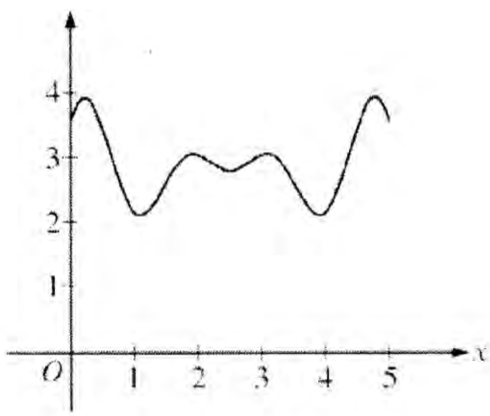
12. For which of the following does $\lim_{x\to\infty} f(x) = 0$?

I. $f(x) = \frac{\ln x}{x^{99}}$

II. $f(x) = \frac{e^x}{\ln x}$

III. $f(x) = \frac{x^{99}}{e^x}$

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) I and III only



Graph of f'

13. The graph of f', the derivative of f, is shown in the figure above. If $f(0) = 20$, which of the following could be the value of $f(5)$?

(A) 15 (B) 20 (C) 25 (D) 35 (E) 40

14. If a and b are positive constants, then $\lim_{x\to\infty} \frac{\ln(bx+1)}{\ln(ax^2+3)} =$

(A) 0 (B) $\frac{1}{2}$ (C) $\frac{1}{2}ab$ (D) 2 (E) ∞

15. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}\left(x + \frac{3}{2}\right)^n$ converges?

(A) $-\frac{5}{2} < x < -\frac{1}{2}$

(B) $-\frac{5}{2} < x \leq -\frac{1}{2}$

(C) $-\frac{5}{2} \leq x < -\frac{1}{2}$

(D) $-\frac{1}{2} < x < \frac{1}{2}$

(E) $x \leq -\frac{1}{2}$

16. For $0 < P < 100$, which of the following is an antiderivative of $\frac{1}{100P - P^2}$?

(A) $\frac{1}{100}\ln(P) - \frac{1}{100}\ln(100 - P)$

(B) $\frac{1}{100}\ln(P) + \frac{1}{100}\ln(100 - P)$

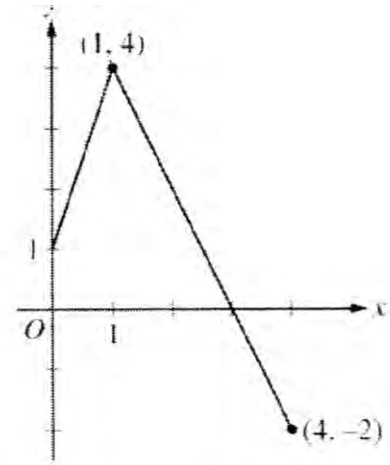
(C) $100\ln(P) - 100\ln(100 - P)$

(D) $\ln\left(100P - P^2\right)$

(E) $\dfrac{1}{50P^2 - \frac{P^3}{3}}$

17. If $\lim_{h \to 0} \frac{\arcsin(a + h) - \arcsin(a)}{h} = 2$, which of the following could be the value of a ?

(A) $\frac{\sqrt{2}}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\sqrt{3}$ (D) $\frac{1}{2}$ (E) 2



Graph of f

18. The graph of the function f, consisting of two line segments, is shown in the figure above. Let g be the function given by $g(x) = 2x + 1$, and let h be the function given by $h(x) = f(g(x))$. What is the value of $h'(1)$?

(A) -4 (B) -2 (C) 4 (D) 6 (E) nonexistent

19. Which of the following is the Maclaurin series for $\dfrac{1}{(1-x)^2}$?

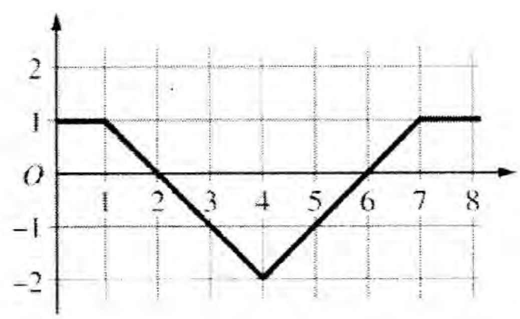
(A) $1 - x + x^2 - x^3 + \cdots$

(B) $1 - 2x + 3x^2 - 4x^3 + \cdots$

(C) $1 + 2x + 3x^2 + 4x^3 + \cdots$

(D) $1 + x^2 + x^4 + x^6 + \cdots$

(E) $x + \dfrac{x^2}{2} + \dfrac{x^3}{3} + \dfrac{x^4}{4} + \cdots$



Graph of f

20. The graph of the function f in the figure above consists of four line segments. Let g be the function defined by $g(x) = \int_0^x f(t)\,dt$. Which of the following is an equation of the line tangent to the graph of g at $x = 5$?

(A) $y + 1 = x - 5$

(B) $y - 2 = x - 5$

(C) $y - 2 = -1(x - 5)$

(D) $y + 2 = x - 5$

(E) $y + 2 = -1(x - 5)$

21. At time $t \geq 0$, a cube has volume $V(t)$ and edges of length $x(t)$. If the volume of the cube decreases at a rate proportional to its surface area, which of the following differential equations could describe the rate at which the volume of the cube decreases?

(A) $\frac{dV}{dt} = -1.2x^2$

(B) $\frac{dV}{dt} = -1.2x^3$

(C) $\frac{dV}{dt} = -1.2x^2t$

(D) $\frac{dV}{dt} = -1.2t^2$

(E) $\frac{dV}{dt} = -1.2V^2$

22. Which of the following is true about the curve $x^2 - xy + y^2 = 3$ at the point $(2, 1)$?

(A) $\frac{dy}{dx}$ exists at $(2, 1)$, but there is no tangent line at that point.

(B) $\frac{dy}{dx}$ exists at $(2, 1)$, and the tangent line at that point is horizontal.

(C) $\frac{dy}{dx}$ exists at $(2, 1)$, and the tangent line at that point is neither horizontal nor vertical.

(D) $\frac{dy}{dx}$ does not exist at $(2, 1)$, and the tangent line at that point is vertical.

(E) $\frac{dy}{dx}$ does not exist at $(2, 1)$, and the tangent line at that point is horizontal.

23. What is the coefficient of x^6 in the Taylor series for $\frac{e^{3x^2}}{2}$ about $x = 0$?

(A) $\frac{1}{1440}$ (B) $\frac{81}{160}$ (C) $\frac{9}{4}$ (D) $\frac{9}{2}$ (E) $\frac{27}{2}$

24. The function g is given by $g(x) = 4x^3 + 3x^2 - 6x + 1$. What is the absolute minimum value of g on the closed interval $[-2, 1]$?

(A) -7 (B) $-\frac{3}{4}$ (C) 0 (D) 2 (E) 6

25. Which of the following is the solution to the differential equation $\frac{dy}{dx} = e^{y+x}$ with the initial condition $y(0) = -\ln 4$?

(A) $y = -x - \ln 4$

(B) $y = x - \ln 4$

(C) $y = -\ln(-e^x + 5)$

(D) $y = -\ln(e^x + 3)$

(E) $y = \ln(e^x + 3)$

26. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{|\sin n|}{n^2}$ II. $\sum_{n=1}^{\infty} e^{-n}$ III. $\sum_{n=1}^{\infty} \frac{n+2}{n^2+n}$

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I and III only

27. If $\int_1^x f(t)\,dt = \frac{20x}{\sqrt{4x^2+21}} - 4$, then $\int_1^\infty f(t)\,dt$ is

(A) 6 (B) 1 (C) -3 (D) -4 (E) divergent

28. If $x = t^2 - 1$ and $y = \ln t$, what is $\frac{d^2y}{dx^2}$ in terms of t ?

(A) $-\frac{1}{2t^4}$ (B) $\frac{1}{2t^4}$ (C) $-\frac{1}{t^3}$ (D) $-\frac{1}{2t^2}$ (E) $\frac{1}{2t^2}$

END OF PART A OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS BC

SECTION I, Part B

Time—50 minutes

Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

29. Let f be a function whose derivative is given by $f'(x) = \ln\left(x^4 + 5x^3 + x^2 - 7x + 28\right)$. On the open interval $(-4, 1)$, at which of the following values of x does f attain a relative maximum?

(A) -3.623 only

(B) -0.871 only

(C) -3.623 and -3.284

(D) -3.459 and 0.581 only

(E) -3.459, -0.871, and 0.581

a	$\lim_{x \to a^-} f(x)$	$\lim_{x \to a^+} f(x)$	$f(a)$
-1	4	6	4
0	-3	-3	5
1	2	2	2

30. The function f has the properties indicated in the table above. Which of the following must be true?

(A) f is continuous at $x = -1$

(B) f is continuous at $x = 0$

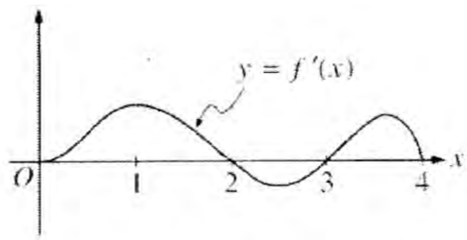
(C) f is continuous at $x = 1$

(D) f is differentiable at $x = 0$

(E) f is differentiable at $x = 1$.

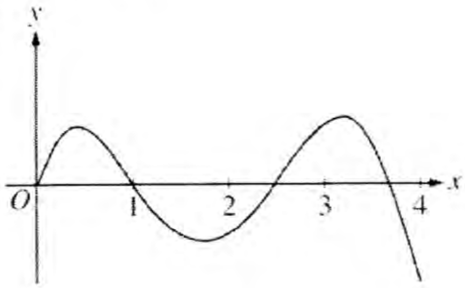
31. What is the area of the region in the first quadrant enclosed by the graphs of $y = \sin(2x)$ and $y = x$

(A) 0.208 (B) 0.210 (C) 0.266 (D) 0.660 (E) 0.835

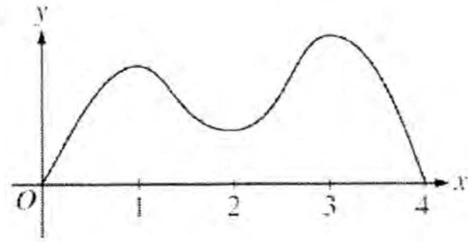


32. The figure above shows the graph of f', the derivative of the function f. If $f(0) = 0$, which of the following could be the graph of f?

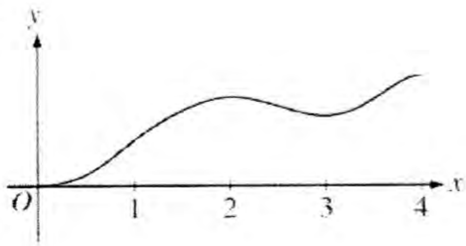
(A)



(B)



(C)



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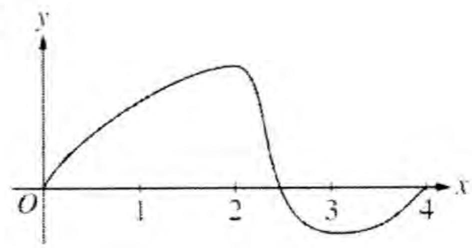
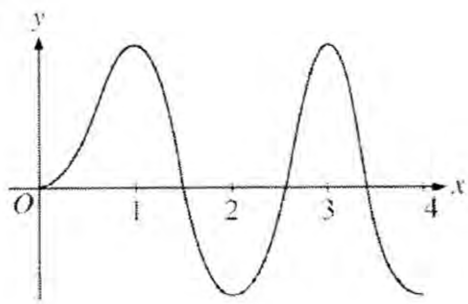


(E)



33. The volume of a certain cone for which the sum of its radius, r, and height is constant is given by $V = \frac{1}{3}\pi r^2 (10 - r)$. The rate of change of the radius of the cone with respect to time is 6. In terms of r, what is the rate of change of the volume of the cone with respect to time?

(A) $-24\pi r$ (B) $6\pi r$ (C) $\frac{20}{3}\pi r - \pi r^2$ (D) $16\pi r - \frac{4}{3}\pi r^2$ (E) $40\pi r - 6\pi r^2$

34. A cup of tea is cooling in a room that has a constant temperature of 70 degrees Fahrenheit (°F). If the initial temperature of the tea, at time $t = 0$ minutes, is 200°F and the temperature of the tea changes at the rate $R(t) = -6.89e^{-0.053t}$ degrees Fahrenheit per minute, what is the temperature, to the nearest degree, of the tea after 4 minutes?

(A) 175°F (B) 130°F (C) 95°F (D) 70°F (E) 45°F

35. Consider the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$, where $a_n > 0$ and $b_n > 0$ for $n \geq 1$. If $\sum_{n=1}^{\infty} a_n$ converges, which of the following must be true?

(A) If $a_n \leq b_n$, then $\sum_{n=1}^{\infty} b_n$ converges.

(B) If $a_n \leq b_n$, then $\sum_{n=1}^{\infty} b_n$ diverges.

(C) If $b_n \leq a_n$, then $\sum_{n=1}^{\infty} b_n$ converges.

(D) If $b_n \leq a_n$, then $\sum_{n=1}^{\infty} b_n$ diverges.

(E) If $b_n \leq a_n$, then the behavior of $\sum_{n=1}^{\infty} b_n$ cannot be determined from the information given.

x	0	0.5	1	1.5	2	2.5	3
$f(x)$	0	4	10	18	28	40	54

36 The table above gives selected values for a continuous function f. If f is increasing over the closed interval $[0, 3]$, which of the following could be the value of $\int_0^3 f(x)\,dx$?

(A) 50 (B) 62 (C) 77 (D) 100 (E) 154

37. Let f be a function with derivative given by $f'(x) = x^3 - 5x^2 + e^x$. On which of the following intervals is the graph of f concave down?

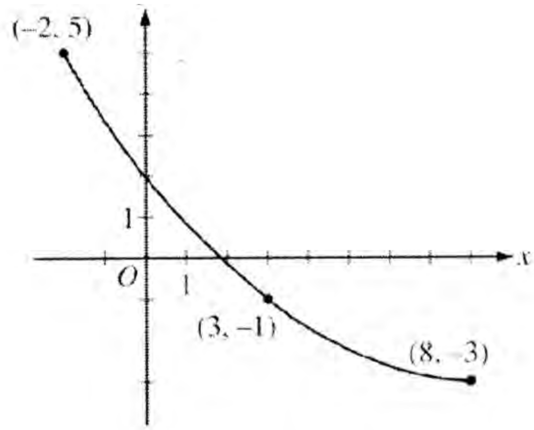
(A) $(-\infty, 0.117)$ only

(B) $(-\infty, 1.144)$

(C) $(0.116, 2.062)$

(D) $(0.673, 2.863)$

(E) $(2.863, \infty)$



Graph of f

38. A portion of the graph of a differentiable function f is shown above. If the value $c = 3$ satisfies the conclusion of the Mean Value Theorem applied to f on the open interval $-2 < x < 8$, what is the slope of the line tangent to the graph of f at $x = 3$?

(A) $-\frac{7}{5}$ (B) $-\frac{5}{4}$ (C) $-\frac{4}{5}$ (D) $-\frac{5}{7}$ (E) $-\frac{1}{5}$

39. If $f'(x) > 0$ for all x and $f''(x) < 0$ for all x, which of the following could be a table of values for f?

(A)

x	$f(x)$
−1	4
0	3
1	1

(B)

x	$f(x)$
−1	4
0	4
1	4

(C)

x	$f(x)$
−1	4
0	5
1	6

(D)

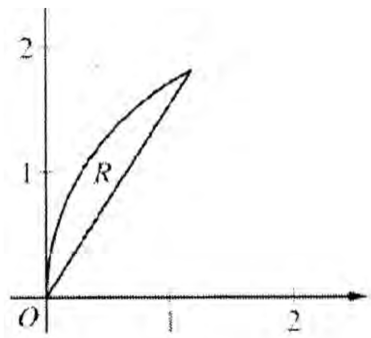
x	$f(x)$
−1	4
0	5
1	7

(E)

x	$f(x)$
−1	4
0	6
1	7

40. The position of a particle moving in the xy-plane is given by the parametric functions $x(t)$ and $y(t)$ for which $x'(t) = t\sin t$ and $y'(t) = 5e^{-3t} + 2$. What is the slope of the line tangent to the path of the particle at the point at which $t = 2$?

(A) 0.904 (B) 1.107 (C) 1.819 (D) 2.012 (E) 3.660



41. Let R be the region in the first quadrant that is bounded above by the polar curve $r = 4\cos\theta$ and below by the line $\theta = 1$, as shown in the figure above. What is the area of R?

(A) 0.317 (B) 0.465 (C) 0.929 (D) 2.618 (E) 5.819

42. What is the volume of the solid generated when the region bounded by the graph of $x = \sqrt{y-2}$ and the lines $x = 0$ and $y = 5$ is revolved about the y-axis?

(A) 3.464 (B) 4.500 (C) 7.854 (D) 10.883 (E) 14.137

43. Which of the following statements are true about the series $\sum_{n=2}^{\infty} a_n$, where $a_n = \dfrac{(-1)^n}{\sqrt{n} + (-1)^n}$?

I. The series is alternating.

II. $|a_{n+1}| \le |a_n|$ for all $n \ge 2$

III. $\lim_{n\to\infty} a_n = 0$

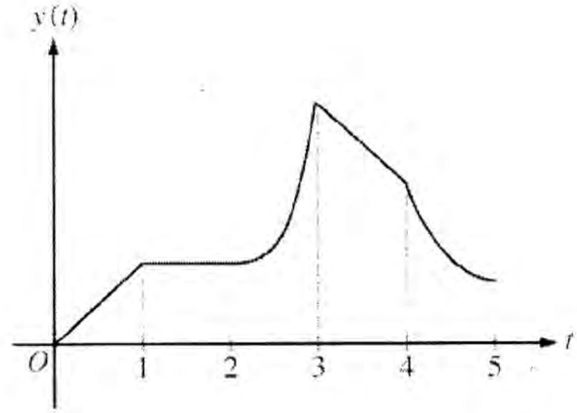
(A) None

(B) I only

(C) I and II only

(D) I and III only

(E) I, II, and III



44. A particle moves along the y-axis. The graph of the particle's position $y(t)$ at time t is shown above for $0 \le t \le 5$. For what values of t is the velocity of the particle negative and the acceleration positive?

(A) $0 < t < 1$ (B) $1 < t < 2$ (C) $2 < t < 3$ (D) $3 < t < 4$ (E) $4 < t < 5$

45. If f is a function such that $f'(x) = -f(x)$, then $\int x f(x)\,dx =$

(A) $f(x)(x+1) + C$

(B) $-f(x)(x+1) + C$

(C) $\frac{x^2}{2} f(x) + C$

(D) $-\frac{x^2}{2} f(x) + C$

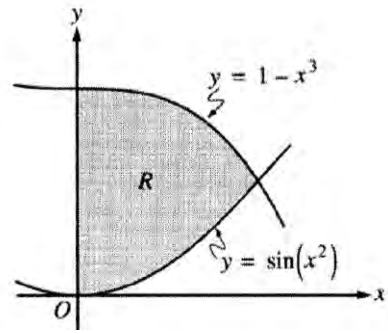
(E) $-\frac{x^2}{2} f(x)\left(1 + \frac{x}{3}\right) + C$

Name ___________________

Periods _____

CALCULUS BC
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

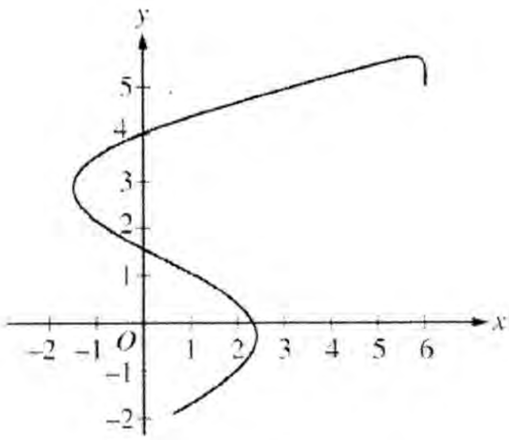


1. Let R be the shaded region in the first quadrant enclosed by the y-axis and the graphs of $y = 1 - x^3$ and $y = \sin\left(x^2\right)$, as shown in the figure above.

(a) Find the area of R.

(b) A horizontal line, $y = k$, is drawn through the point where the graphs of $y = 1 - x^3$ and $y = \sin\left(x^2\right)$ intersect. Find k and determine whether this line divides R into two regions of equal area. Show the work that leads to your conclusion.

(c) Find the volume of the solid generated when R is revolved about the line $y = -3$.



2. A planetary rover travels on a flat surface. The path of the rover for the time interval $0 \le t \le 2$ hours is shown in the rectangular coordinate system above. The rover starts at the point with coordinates $(6, 5)$ at time $t = 0$. The coordinates $(x(t), y(t))$ of the position of the rover change at rates given by

$$x'(t) = -12\sin\left(2t^2\right)$$
$$y'(t) = 10\cos\left(1 + \sqrt{t}\right),$$

where $x(t)$ and $y(t)$ are measured in meters and t is measured in hours.

(a) Find the acceleration vector of the rover at time $t = 1$. Find the speed of the rover at time $t = 1$.

(b) Find the total distance that the rover travels over the time interval $0 \le t \le 1$.

(c) Find the y-coordinate of the position of the rover at time $t = 1$.

(d) The rover receives a signal at each point where the line tangent to its path has slope $\frac{1}{2}$. At what times t, for $0 \le t \le 2$, does the rover receive a signal?

Do not go on to Part B until you are told to do so. You will be able to work more on these first two problems but you will not be allowed to use your calculator.

CALCULUS BC
SECTION II, Part B
No calculator is allowed for these problems.

Time—60 minutes
Number of problems—4

t (days)	0	10	22	30
$W'(t)$ (GL per day)	0.6	0.7	1.0	0.5

3. The twice-differentiable function W models the volume of water in a reservoir at time t, where $W(t)$ is measured in gigaliters (GL) and t is measured in days. The table above gives values of $W'(t)$ sampled at various times during the time interval $0 \le t \le 30$ days. At time $t = 30$, the reservoir contains 125 gigaliters of water.

   (a) Use the tangent line approximation to W at time $t = 30$ to predict the volume of water $W(t)$, in gigaliters, in the reservoir at time $t = 32$. Show the computations that lead to your answer.

   (b) Use a left Riemann sum, with the three subintervals indicated by the data in the table, to approximate $\int_0^{30} W'(t)\,dt$. Use this approximation to estimate the volume of water $W(t)$, in gigaliters, in the reservoir at time $t = 0$. Show the computations that lead to your answer.

   (c) Explain why there must be at least one time t, other than $t = 10$, such that $W'(t) = 0.7$ GL/day.

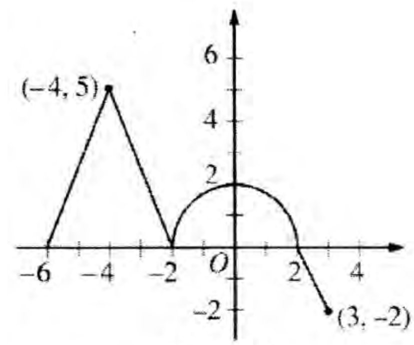
   (d) The equation $A = 0.3W^{2/3}$ gives the relationship between the area A, in square kilometers, of the surface of the reservoir, and the volume of water $W(t)$, in gigaliters, in the reservoir. Find the instantaneous rate of change of A, in square kilometers per day, with respect to t when $t = 30$ days.

4. Consider the function f given by $f(x) = xe^{-x^2}$ for all real numbers x.

(a) At what value of x does $f(x)$ attain its absolute maximum? Justify your answer.

(b) Find an antiderivative of f.

(c) Find the value of $\int_0^{\infty} xf(x)\,dx$, given the fact that $\int_0^{\infty} e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2}$.



Graph of f

5. The graph of the continuous function f, consisting of three line segments and a semicircle, is shown above. Let g be the function given by $g(x) = \int_{-2}^{x} f(t)\,dt$.

(a) Find $g(-6)$ and $g(3)$.

(b) Find $g'(0)$.

(c) Find all values of x on the open interval $-6 < x < 3$ for which the graph of g has a horizontal tangent. Determine whether g has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

(d) Find all values of x on the open interval $-6 < x < 3$ for which the graph of g has a point of inflection. Explain your reasoning.

6. The function f satisfies the equation

$$f'(x) = f(x) + x + 1$$

and $f(0) = 2$. The Taylor series for f about $x = 0$ converges to $f(x)$ for all x.

(a) Write an equation for the line tangent to the curve $y = f(x)$ at the point where $x = 0$.

(b) Find $f''(0)$ and find the second-degree Taylor polynomial for f about $x = 0$.

(c) Find the fourth-degree Taylor polynomial for f about $x = 0$.

(d) Find $f^{(n)}(0)$, the nth derivative of f at $x = 0$, for $n \geq 2$. Use the Taylor series for f about $x = 0$ and the Taylor series for e^x about $x = 0$ to find a polynomial expression for $f(x) - 4e^x$.

Answer Key for AP Calculus BC Practice Exam, Section I

Question 1: C
Question 2: C
Question 3: D
Question 4: E
Question 5: A
Question 6: D
Question 7: C
Question 8: D
Question 9: D
Question 10: D
Question 11: D
Question 12: E
Question 13: D
Question 14: B
Question 15: B
Question 16: A
Question 17: B
Question 18: A
Question 19: C
Question 20: E
Question 21: A
Question 22: D
Question 23: C
Question 24: A
Question 25: C
Question 26: D
Question 27: A
Question 28: A
Question 29: A
Question 30: C
Question 31: B
Question 32: C
Question 33: E
Question 34: A
Question 35: C
Question 36: B
Question 37: C
Question 38: C
Question 39: E
Question 40: B
Question 41: B
Question 42: E
Question 43: D
Question 44: E
Question 45: B

2013 BC MC Solutions

1. $f'(x) = \frac{(x+3)(2x+3)-(x^2+3x+2)}{(x+3)^2}$

$= \frac{2x^2+3x+6x+9-x^2-3x-2}{(x+3)^2}$

$= \frac{x^2+6x+7}{(x+3)^2}$ [C]

2. $\int \left(5x^{\frac{3}{2}} - 5x^3\right)dx$

$5\cdot\frac{2}{5}x^{5/2} - \frac{5}{4}x^4 + C$

$2x^{\frac{5}{2}} - \frac{5}{4}x^4 + C$ [C]

3. $\sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{5^n} = \frac{9}{5} - \frac{27}{25} + \cdots$

$a_0 = \frac{9}{5}$, $r = \frac{-3}{5}$

$S = \frac{a_0}{1-r}$

$= \frac{\frac{9}{5}}{1+\frac{3}{5}}$

$= \frac{9}{5}\cdot\frac{5}{8}$ [D]

$= \frac{9}{8}$

4. $x^2 - 3xy = 10$

$2x - 3xy' + y(-3) = 0$

$-3xy' = 3y - 2x$

$y' = \frac{3y-2x}{-3x}$

$y'\big|_{(1,-3)} = \frac{-9-2}{-3}$

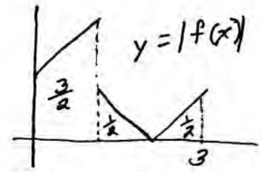
$= \frac{11}{3}$

T.L. $y+3 = \frac{11}{3}(x-1)$ [E]

5. $y = \frac{1}{2}x^{\frac{4}{5}} - 3x^{-5}$

$y' = \frac{2}{5}x^{-\frac{1}{5}} + 15x^{-6}$ [A]

6.



$\int_0^3 |f(x)|\,dx = \frac{3}{2}+\frac{1}{2}+\frac{1}{2}$

$= \frac{5}{2}$ [D]

7. Increasing @ decreasing rate means concave down. On a logistic curve this is from half the carrying capacity to the carrying capacity. $500 < y < 1000$

[C]

8. $\frac{dx}{dt} = 3$ $\quad \frac{dy}{dt} = 2t$

arc length $= \int_0^1 \sqrt{3^2 + (2t)^2}\,dt$

[D]

9.

x	y	$\frac{dy}{dx} = 2x+y$	$\Delta y = m\,\Delta x$
1	0	2	$\Delta y = 2\cdot\frac{1}{2} = 1$
$\frac{3}{2}$	1	$3+1=4$	$\Delta y = 4\cdot\frac{1}{2} = 2$
2	3		

$f(2) \approx 3$ [D]

10. $\frac{1}{2}\int_0^k \frac{2x}{x^2+4}\,dx = \frac{1}{2}\ln 4$

$\frac{1}{2}\ln(x^2+4)\Big|_0^k = \frac{1}{2}\ln 4$

$\frac{1}{2}\ln(k^2+4) - \frac{1}{2}\ln 4 = \frac{1}{2}\ln 4$

$\frac{1}{2}\ln(k^2+4) = \ln 4$

$\ln(k^2+4) = 2\ln 4$

$\ln(k^2+4) = \ln 16$

$k^2+4 = 16$

$k^2 = 12$

$k = \sqrt{12}$ [D]

11. $\frac{f'''(4)}{3!}(x-4)^3 = \frac{(x-4)^3}{512}$

$f'''(4) = \frac{3!}{512}$

$= \frac{3}{256}$ [D]

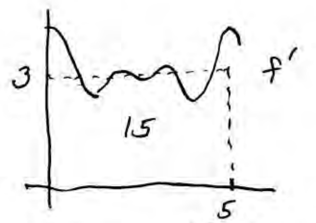
12. I $\lim_{x\to\infty} \frac{\ln x}{x^{99}} = 0$ True

II $\lim_{x\to\infty} \frac{e^x}{\ln x} = \infty$ False

III $\lim_{x\to\infty} \frac{x^{99}}{e^x} = 0$ True

[E]

13.



$f(5) = 20 + \int_0^5 f'(x)\,dx$

$\approx 20 + 15$

$= 35$ [D]

14. $\lim_{x\to\infty} \frac{\ln(bx+1)}{\ln(ax^2+3)}$ $\frac{\infty}{\infty}$ form

$\lim_{x\to\infty} \frac{\frac{b}{bx+1}}{\frac{2ax}{ax^2+3}}$

$\lim_{x\to\infty} \frac{b(ax^2+3)}{2ax(bx+1)}$

$\lim_{x\to\infty} \frac{abx^2+3b}{2abx^2+2ax}$

$\frac{1}{2}$ [B]

15. Ratio Test:

$\lim_{n\to\infty} \left|\frac{(x+\frac{3}{2})^{n+1}}{n+1} \cdot \frac{n}{(x+\frac{3}{2})^n}\right|$

$|x+\frac{3}{2}| < 1$

$-1 < x+\frac{3}{2} < 1$

$-\frac{5}{2} < x < -\frac{1}{2}$

check $x = -\frac{5}{2}$: $\sum \frac{(-1)^n}{n}(-1)^n$, $\sum \frac{1}{n}$ harmonic, div.

check $x = -\frac{1}{2}$: $\sum \frac{(-1)^n}{n}(1)^n$, $\sum \frac{(-1)^n}{n}$ alt. harm. conv.

IOC: $\left(-\frac{5}{2}, -\frac{1}{2}\right]$ [B]

16. $\int \frac{1}{P(100-P)}\,dP$

$\frac{A}{P} + \frac{B}{100-P} = \frac{1}{P(100-P)}$

$A(100-P) + BP = 1$

$P = 100 \longrightarrow B\cdot 100 = 1$, $B = \frac{1}{100}$

$P = 0 \longrightarrow A\cdot 100 = 1$, $A = \frac{1}{100}$

$\frac{1}{100}\int \frac{1}{P}\,dP - \frac{1}{100}\int \frac{-1}{100-P}\,dP$

$\frac{1}{100}\ln P - \frac{1}{100}\ln(100-P) + C$

[A]

17. This is the limit def. of the derivative of arcsin x evaluated at $x = a$.

$\frac{d}{dx}\arcsin x\Big|_{x=a} = 2$

$\frac{1}{\sqrt{1-x^2}}\Big|_{x=a} = 2$ (could use L'Hop.)

$\frac{1}{\sqrt{1-a^2}} = 2$ [B]

$\sqrt{1-a^2} = \frac{1}{2}$

$1-a^2 = \frac{1}{4}$

$\frac{3}{4} = a^2 \longrightarrow a = \frac{\sqrt{3}}{2}$

18. $h(x) = f(2x+1)$

$h'(x) = 2f'(2x+1)$

$h'(1) = 2f'(3)$

$= 2 \cdot (-2)$ ← slope of f @ $x=3$

$= -4$ [A]

19. $f(x) = (1-x)^{-2}$ $f(0) = 1$

$f'(x) = -2(1-x)^{-3}(-1)$ $f'(0) = 2$

$= 2(1-x)^{-3}$

no need to do more

$f(x) = 1 + 2x + \ldots$ [C]

20. $g(5) = \int_0^5 f(t)\,dt$

$= \frac{3}{2} - 2 - \frac{3}{2}$ (areas of trapezoids and triangles)

$= -2$

$g'(x) = f(x)$

$g'(5) = f(5)$

$= -1$

$y + 2 = -1(x-5)$ [E]

21. $SA = 6x^2$

Vol. decr. @ rate prop. to SA

$\frac{dv}{dt} = k \cdot 6x^2$ where k is neg.

[A]

22. $x^2 - xy + y^2 = 3$

$2x - xy' + y(-1) + 2yy' = 0$

$y'(-x+2y) = y - 2x$

$y' = \frac{y-2x}{2y-x}$

$y'|_{(2,1)} = \frac{-3}{0}$

[D]

23. $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \ldots$

$\frac{e^{3x^2}}{2} = \frac{1 + 3x^2 + \frac{9x^4}{2} + \frac{27x^6}{6} + \ldots}{2}$

coeff. of $x^6 = \frac{27}{6} \cdot \frac{1}{2} = \frac{9}{4}$

[C]

24. $g'(x) = 12x^2 + 6x - 6$

$0 = 6(2x^2 + x - 1)$

$0 = 6(2x-1)(x+1)$

CN $x = \frac{1}{2}, -1$

candidate test

$g(-2) = -32 + 12 + 12 + 1 = -7$

$g(-1) = -4 + 3 + 6 + 1 = 6$

$g(\frac{1}{2}) = \frac{1}{2} + \frac{3}{4} - 3 + 1 = -\frac{3}{4}$

$g(1) = 4 + 3 - 6 + 1 = 2$

min. $g = -7$ [A]

25. It is impossible to separate and integrate. Differentiate the choices. $y'|_{(0,-\ln 4)} = e^{0-\ln 4} = e^{\ln 4^{-1}} = \frac{1}{4}$

(A) $y' = -1$ no

(B) $y' = 1$ no

(C) $y' = \frac{e^x}{-e^x+5}$

$y'|_{x=0} = \frac{1}{-1+5} = \frac{1}{4}$ yes and $-\ln 4 = -\ln(1+3)$

(D) $y' = \frac{-e^x}{e^x+3}$

$y'|_{x=0} = \frac{-1}{1+3}$ no [C]

(E) $y' = \frac{e^x}{e^x+3}$

$y'|_{x=0} = \frac{1}{1+3}$ yes but

$-\ln 4 \neq \ln(e^0+3)$

26. I Compare to $\sum \frac{1}{n^2}$, $p=2$ conv,

$\frac{|\sin n|}{n^2} < \frac{1}{n^2}$, $\sum \frac{|\sin n|}{n^2}$ conv. by DCT

II $\sum \frac{1}{e^n}$

$\lim_{n\to\infty} \frac{1}{e^{n+1}} \cdot \frac{e^n}{1}$

$\frac{1}{e} < 1$ conv. by RaT

III Compare to $\sum \frac{1}{n}$ div. harmonic

$\lim_{n\to\infty} \frac{n+2}{n^2+n} \cdot \frac{n}{1} = 1$

$\sum \frac{n+2}{n^2+n}$ also div. by LCT

D

27. $\int_1^\infty f(t)\,dt$

$\lim_{b\to\infty} \int_1^b f(t)\,dt$

$\lim_{b\to\infty} \left(\frac{20b}{\sqrt{4b^2+21}} - 4\right) - \left(\frac{20\cdot 1}{\sqrt{4+21}} - 4\right)$

$\left(\frac{20}{2} - 4\right) - (4-4)$

$10 - 4$

6 A

28. $\frac{dx}{dt} = 2t \quad \frac{dy}{dt} = \frac{1}{t}$

$\frac{dy}{dx} = \frac{\frac{1}{t}}{2t}$

$= \frac{1}{2t^2}$

$= \frac{1}{2}t^{-2}$

$\frac{d^2y}{dx^2} = \frac{-t^{-3}}{2t}$ A

$= -\frac{1}{2t^4}$

29. Graph $f'(x)$

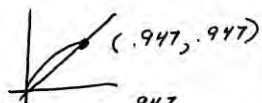
f has a rel. max. when f' changes from pos. to neg.

This happens only once at about -3.5 A

30. C because the limit exists and is equal to the value of the function.

f has a jump @ $x=-1$ and a hole @ $x=0$

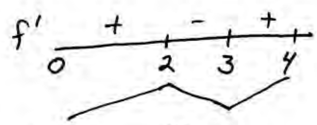
31.



$A = \int_0^{.947} (\sin(2x) - x)\,dx$

$= .210$ B

32.



rel. max. @ $x=2$

rel. min. @ $x=3$

Since $\int_0^3 f'(x)\,dx > 0$

$f(3) > 0$ C

33. $V = \frac{10}{3}\pi r^2 - \frac{1}{3}\pi r^3 \quad \frac{dr}{dt} = 6$

$\frac{dV}{dt} = \frac{20}{3}\pi r\frac{dr}{dt} - \pi r^2\frac{dr}{dt}$

$= \frac{20}{3}\pi r(6) - \pi r^2(6)$

$= 40\pi r - 6\pi r^2$ E

34. $\text{Temp}(4) = \text{Temp}(0) + \int_0^4 -6.89e^{-.053t}\,dt$

$\approx 200 - 25$ A

$= 175$

35. C DCT

36. If f is incr.

Left Riemann Sum $< \int_0^3 f(x)\,dx <$ Right RS

$\frac{1}{2}(0+4+10+18+28+40) < \int_0^3 f(x)\,dx <$

$\frac{1}{2}(4+10+18+28+40+54)$

$50 < \int_0^3 f(x)\,dx < 77$ B

37. $f'(x) = x^3 - 5x^2 + e^x$

$f''(x) = 3x^2 - 10x + e^x < 0$

graph to see where f'' is neg

C

38. IROC = AROC

$f'(3) = \frac{f(8) - f(-2)}{8 - (-2)}$

$= \frac{-3-5}{10}$

$= -\frac{4}{5}$ C

39. $f'(x) > 0 \rightarrow f$ is incr.

(could be C, D, or E)

$f''(x) < 0 \rightarrow f$ is concave down

(Incr. @ decr. rate)

E

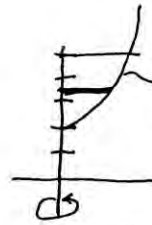
40. $\left.\frac{dy}{dx}\right|_{t=2} = \frac{y'(2)}{x'(2}$

$= 1.107$ B

41. $A = \frac{1}{2}\int_1^{\frac{\pi}{2}} (4\cos\theta)^2\,d\theta$ $\quad 4\cos\theta = 0$, $\theta = \frac{\pi}{2}$

$= .465$ B

42.



disc method

$r = \sqrt{y-2}$

$V = \pi \int_2^5 (\sqrt{y-2})^2\,dy$

$= 14.137$ E

43. $\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n}+(-1)^n} = \frac{1}{\sqrt{2}+1} - \frac{1}{\sqrt{3}-1} + \frac{1}{\sqrt{4}+1} - \cdots$

I alt. series True

II not decr. False

III $\lim_{n\to\infty} \frac{(-1)^n}{\sqrt{n}+(-1)^n} = 0$ True

D

44. $v(t) < 0 \rightarrow y(t)$ is decr.

$a(t) > 0 \rightarrow y(t)$ is ccu

E

45. $\int x f(x)\,dx = \int -x f'(x)\,dx$

let $u = -x$

$du = -1\,dx$

$dv = f'(x)dx$

$v = f(x)$

$\int x f(x)\,dx = -x f(x) - \int f(x)(-1)\,dx$

$= -x f(x) - \int f'(x)\,dx$

$= -x f(x) - f(x) + C$

$= -f(x)(x+1) + C$

B

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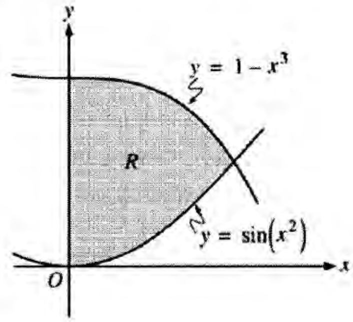
Question 1

Let R be the shaded region in the first quadrant enclosed by the y-axis and the graphs of $y = 1 - x^3$ and $y = \sin(x^2)$, as shown in the figure above.



(a) Find the area of R.

(b) A horizontal line, $y = k$, is drawn through the point where the graphs of $y = 1 - x^3$ and $y = \sin(x^2)$ intersect. Find k and determine whether this line divides R into two regions of equal area. Show the work that leads to your conclusion.

(c) Find the volume of the solid generated when R is revolved about the line $y = -3$.

The graphs of $y = 1 - x^3$ and $y = \sin(x^2)$ intersect in the first quadrant at the point $(A, B) = (0.764972, 0.552352)$.	1 : correct limits in an integral in (a), (b), or (c)
(a) Area $= \int_0^A \left(1 - x^3 - \sin(x^2)\right) dx$ $= 0.533$ (or 0.534)	2 : { 1 : integrand 1 : answer
(b) $k = B = 0.552352$ $\int_0^A \left(1 - x^3 - k\right) dx = 0.257$ (or 0.256) $\int_0^A \left(k - \sin(x^2)\right) dx = 0.277$ (or 0.276) The two regions have unequal areas.	3 : { 1 : integral(s) with k value 1 : value(s) of integral(s) 1 : conclusion tied to part (a) or comparison of two integrals Note: Stating k value only does not earn a point.
(c) Volume $= \pi \int_0^A \left(\left(1 - x^3 + 3\right)^2 - \left(\sin(x^2) + 3\right)^2\right) dx$ $= 11.841$ (or 11.840)	3 : { 2 : integrand 1 : answer

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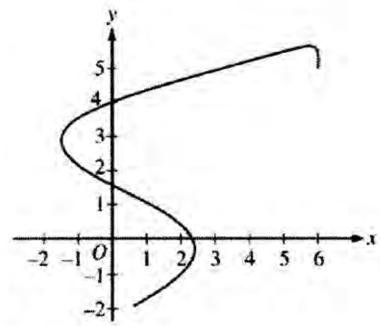
Question 2



A planetary rover travels on a flat surface. The path of the rover for the time interval $0 \le t \le 2$ hours is shown in the rectangular coordinate system above. The rover starts at the point with coordinates $(6, 5)$ at time $t = 0$. The coordinates $(x(t), y(t))$ of the position of the rover change at rates given by

$$x'(t) = -12\sin\left(2t^2\right)$$
$$y'(t) = 10\cos\left(1 + \sqrt{t}\right),$$

where $x(t)$ and $y(t)$ are measured in meters and t is measured in hours.

(a) Find the acceleration vector of the rover at time $t = 1$. Find the speed of the rover at time $t = 1$.

(b) Find the total distance that the rover travels over the time interval $0 \le t \le 1$.

(c) Find the y-coordinate of the position of the rover at time $t = 1$.

(d) The rover receives a signal at each point where the line tangent to its path has slope $\frac{1}{2}$. At what times t, for $0 \le t \le 2$, does the rover receive a signal?

(a) $a(1) = \langle x''(1), y''(1)\rangle \quad \langle 19.975, -4.546\rangle$ Speed $= \sqrt{(x'(1))^2 + (y'(1))^2} = 11.678$	$2 : \begin{cases} 1 : \text{acceleration vector} \\ 1 : \text{speed} \end{cases}$
(b) Distance $= \int_0^1 \sqrt{(x'(t))^2 + (y'(t))^2}\, dt = 6.704$ (or 6.703)	$3 : \begin{cases} 2 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(c) $y(1) = 5 + \int_0^1 y'(t)\, dt = 4.057$ (or 4.056)	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(d) $\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{10\cos\left(1 + \sqrt{t}\right)}{-12\sin\left(2t^2\right)} = \frac{1}{2}$ $t = 1.072$	$2 : \begin{cases} 1 : \text{equation} \\ 1 : \text{answer} \end{cases}$

Question 3

t (days)	0	10	22	30
$W'(t)$ (GL per day)	0.6	0.7	1.0	0.5

The twice-differentiable function W models the volume of water in a reservoir at time t, where $W(t)$ is measured in gigaliters (GL) and t is measured in days. The table above gives values of $W'(t)$ sampled at various times during the time interval $0 \le t \le 30$ days. At time $t = 30$, the reservoir contains 125 gigaliters of water.

(a) Use the tangent line approximation to W at time $t = 30$ to predict the volume of water $W(t)$, in gigaliters, in the reservoir at time $t = 32$. Show the computations that lead to your answer.

(b) Use a left Riemann sum, with the three subintervals indicated by the data in the table, to approximate $\int_0^{30} W'(t)\,dt$. Use this approximation to estimate the volume of water $W(t)$, in gigaliters, in the reservoir at time $t = 0$. Show the computations that lead to your answer.

(c) Explain why there must be at least one time t, other than $t = 10$, such that $W'(t) = 0.7$ GL/day.

(d) The equation $A = 0.3W^{2/3}$ gives the relationship between the area A, in square kilometers, of the surface of the reservoir, and the volume of water $W(t)$, in gigaliters, in the reservoir. Find the instantaneous rate of change of A, in square kilometers per day, with respect to t when $t = 30$ days.

(a) An equation of the tangent line is $y = 0.5(t - 30) + 125$.
$W(32) \approx 0.5(32 - 30) + 125 = 126$

1 : answer

(b) $\int_0^{30} W'(t)\,dt \approx (10)(0.6) + (12)(0.7) + (8)(1.0) = 22.4$

$W(0) = W(30) - \int_0^{30} W'(t)\,dt = 125 - 22.4 = 102.6$

$3 : \begin{cases} 1 : \text{left Riemann sum} \\ 1 : \text{approximation} \\ 1 : \text{answer} \end{cases}$

(c) W' is differentiable $\Rightarrow$ W' is continuous.

$W'(30) = 0.5 < 0.7 < 1.0 = W'(22)$

By the Intermediate Value Theorem, there must be at least one time t, $22 \le t \le 30$, such that $W'(t) = 0.7$.

2 : explanation

(d) $\frac{dA}{dt} = (0.3)\frac{2}{3}W^{-1/3} \cdot \frac{dW}{dt} = \frac{0.2}{\sqrt[3]{W}} \cdot \frac{dW}{dt}$

$\left.\frac{dA}{dt}\right|_{t=30} = \frac{0.2}{\sqrt[3]{125}} \cdot 0.5 = 0.02$

$3 : \begin{cases} 2 : \frac{dA}{dt} \\ 1 : \text{answer} \end{cases}$

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Question 4

$f(x) = xe^{-x^2}$ for all real numbers x.

(a) At what value of x does $f(x)$ attain its absolute maximum? Justify your answer.

(b) Find an antiderivative of f.

(c) Find the value of $\int_0^\infty x f(x)\, dx$, given the fact that $\int_0^\infty e^{-x^2}\, dx = \frac{\sqrt{\pi}}{2}$.

(a) $f'(x) = e^{-x^2} - 2x^2 e^{-x^2} = \left(1 - 2x^2\right)e^{-x^2}$

$f'(x) = 0$ when $x = -\frac{1}{\sqrt{2}}$ or $x = \frac{1}{\sqrt{2}}$

$f(x) < 0$ for $x < 0$, and $f(x) > 0$ for $x > 0$. Therefore, a maximum can only occur at a positive x-value.

$f'(x)$ changes sign from positive to negative at $x = \frac{1}{\sqrt{2}}$, so f has a relative maximum at $x = \frac{1}{\sqrt{2}}$.

f increases on $0 < x \le \frac{1}{\sqrt{2}}$ and decreases on $\frac{1}{\sqrt{2}} \le x < \infty$.

Therefore, there is an absolute maximum at $x = \frac{1}{\sqrt{2}}$.

5 : { 2 : $f'(x)$; 1 : solves $f'(x) = 0$; 1 : answer; 1 : justification }

(b) $\int xe^{-x^2}\, dx = -\frac{1}{2}e^{-x^2} + C$

1 : antiderivative

(c) Using integration by parts,

$u = x \quad dv = xe^{-x^2}\, dx$

$du = dx \quad v = -\frac{1}{2}e^{-x^2}$

$$\int x f(x)\, dx = \int x^2 e^{-x^2}\, dx = -\frac{1}{2}xe^{-x^2} + \frac{1}{2}\int e^{-x^2}\, dx$$

Therefore,

$$\begin{aligned}\int_0^\infty x f(x)\, dx &= \lim_{b\to\infty}\left[-\frac{1}{2}xe^{-x^2}\Big|_0^b + \frac{1}{2}\int_0^b e^{-x^2}\, dx\right]\\ &= \lim_{b\to\infty}\left(-\frac{1}{2}be^{-b^2}\right) + \frac{1}{2}\int_0^\infty e^{-x^2}\, dx\\ &= 0 + \frac{1}{2}\cdot\frac{\sqrt{\pi}}{2} = \frac{\sqrt{\pi}}{4}\end{aligned}$$

3 : { 2 : integration by parts; 1 : answer }

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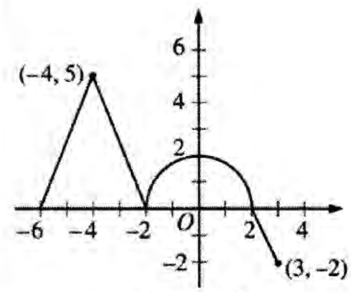
Question 5

The graph of the continuous function f, consisting of three line segments and a semicircle, is shown above. Let g be the function given by $g(x) = \int_{-2}^{x} f(t)\,dt$.



Graph of f

(a) Find $g(-6)$ and $g(3)$.

(b) Find $g'(0)$.

(c) Find all values of x on the open interval $-6 < x < 3$ for which the graph of g has a horizontal tangent. Determine whether g has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

(d) Find all values of x on the open interval $-6 < x < 3$ for which the graph of g has a point of inflection. Explain your reasoning.

(a) $g(-6) = \int_{-2}^{-6} f(t)\,dt = -\int_{-6}^{-2} f(t)\,dt = -\frac{1}{2}\cdot 4\cdot 5 = -10$

$g(3) = \int_{-2}^{3} f(t)\,dt = \frac{1}{2}\pi\cdot 2^2 - \frac{1}{2}\cdot 1\cdot 2 = 2\pi - 1$

$2 : \begin{cases} 1 : g(-6) \\ 1 : g(3) \end{cases}$

(b) $g'(0) = f(0) = 2$

$1 : g'(0)$

(c) The graph of g has a horizontal tangent at $x = -2$ and $x = 2$ where $g'(x) = f(x) = 0$.

The graph of g has neither a local maximum nor a local minimum at $x = -2$ because $g'(x) = f(x)$ does not change sign at $x = -2$.

The graph of g has a local maximum at $x = 2$ because $g'(x) = f(x)$ changes sign from positive to negative at $x = 2$.

$3 : \begin{cases} 1 : \text{horizontal tangent at } x = -2 \\ \quad\ \text{and } x = 2 \\ 2 : \text{answers with justifications} \end{cases}$

(d) The graph of g has a point of inflection at $x = -4$, $x = -2$, and $x = 0$.
$g'(x) = f(x)$ changes from increasing to decreasing at $x = -4$ and $x = 0$, and changes from decreasing to increasing at $x = -2$.

OR

$g''(x) = f'(x)$ changes from positive to negative at $x = -4$ and $x = 0$, and changes from negative to positive at $x = -2$.

$3 : \begin{cases} 2 : \text{values of } x \\ 1 : \text{explanation} \end{cases}$

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Question 6

The function f satisfies the equation

$$f'(x) = f(x) + x + 1$$

and $f(0) = 2$. The Taylor series for f about $x = 0$ converges to $f(x)$ for all x.

(a) Write an equation for the line tangent to the curve $y = f(x)$ at the point where $x = 0$.

(b) Find $f''(0)$ and find the second-degree Taylor polynomial for f about $x = 0$.

(c) Find the fourth-degree Taylor polynomial for f about $x = 0$.

(d) Find $f^{(n)}(0)$, the nth derivative of f at $x = 0$, for $n \geq 2$. Use the Taylor series for f about $x = 0$ and the Taylor series for e^x about $x = 0$ to find a polynomial expression for $f(x) - 4e^x$.

(a) $f'(0) = f(0) + 0 + 1 = 3$

An equation for the tangent line is $y = 3x + 2$.

1 : tangent line equation

(b) $f''(x) = f'(x) + 1;\ f''(0) = f'(0) + 1 = 3 + 1 = 4$

$$P_2(x) = 2 + 3x + \frac{4}{2!}x^2 = 2 + 3x + 2x^2$$

$2 : \begin{cases} 1 : f''(0) \\ 1 : \text{second-degree Taylor polynomial} \end{cases}$

(c) $f'''(x) = f''(x);\ f'''(0) = f''(0) = 4$

$f^{(4)}(x) = f'''(x);\ f^{(4)}(0) = f'''(0) = 4$

$$P_4(x) = 2 + 3x + \frac{4}{2!}x^2 + \frac{4}{3!}x^3 + \frac{4}{4!}x^4$$
$$= 2 + 3x + 2x^2 + \frac{2}{3}x^3 + \frac{1}{6}x^4$$

$2 : \begin{cases} 1 : f'''(0) \text{ and } f^{(4)}(0) \\ 1 : \text{fourth-degree Taylor polynomial} \end{cases}$

(d) $f^{(n)}(0) = 4$ for $n \geq 2$

$$f(x) = 2 + 3x + \frac{4}{2!}x^2 + \frac{4}{3!}x^3 + \frac{4}{4!}x^4 + \frac{4}{5!}x^5 + \cdots$$
$$4e^x = 4 + 4x + \frac{4}{2!}x^2 + \frac{4}{3!}x^3 + \frac{4}{4!}x^4 + \frac{4}{5!}x^5 + \cdots$$

Therefore, $f(x) - 4e^x = -2 - x$.

$4 : \begin{cases} 1 : f^{(n)}(0) \text{ for } n \geq 2 \\ 1 : \text{Taylor series for } f \\ 1 : \text{Taylor series for } e^x \\ 1 : \text{polynomial expression} \end{cases}$

2013 AP Calculus BC Scoring Worksheet

Section I: Multiple Choice

__________ × 1.2000 = __________
Number Correct (out of 45) — Weighted Section I Score (Do not round)

Section II: Free Response

Question 1	_____ (out of 9)	× 1.0000 =	_____ (Do not round)
Question 2	_____ (out of 9)	× 1.0000 =	_____ (Do not round)
Question 3	_____ (out of 9)	× 1.0000 =	_____ (Do not round)
Question 4 (AB Part)	_____ (out of 6)	× 1.0000 =	_____ (Do not round)
Question 4 (BC Part)	_____ (out of 3)	× 1.0000 =	_____ (Do not round)
Question 5	_____ (out of 9)	× 1.0000 =	_____ (Do not round)
Question 6	_____ (out of 9)	× 1.0000 =	_____ (Do not round)

Sum = __________
Weighted Section II Score (Do not round)

Composite Score

__________ + __________ = __________
Weighted Section I Score + Weighted Section II Score = Composite Score (Round to nearest whole number)

AP Score Conversion Chart
Calculus BC

Composite Score Range	AP Score
66-108	5
57-65	4
44-56	3
39-43	2
0-38	1