CALCULUS BC
SECTION I, Part A
Time—55 minutes
Number of questions—28

Name ____________________

Periods________

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

1. $\int \frac{x^3+5}{x^2}\,dx =$

(A) $1 - \frac{10}{x^3} + C$

(B) $\frac{3x}{4} + \frac{15}{x^2} + C$

(C) $\frac{x^2}{2} - \frac{5}{x} + C$

(D) $\frac{x^2}{2} - \frac{5}{3x^3} + C$

(E) $-\frac{x^3}{4} - 5 + C$

2. What is the slope of the line tangent to the graph of $y = \ln(2x)$ at the point where $x = 4$?

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$ (E) 4

3. $\lim_{x \to 0} \frac{x^2}{1 - \cos x}$ is

(A) -2 (B) 0 (C) 1 (D) 2 (E) nonexistent

4. $\int \frac{1}{x^2 - 7x + 10} dx =$

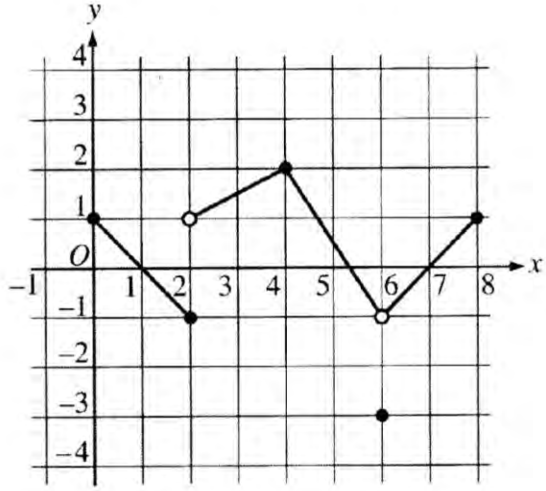
(A) $\ln|(x-2)(x-5)| + C$

(B) $\frac{1}{3}\ln|(x-2)(x-5)| + C$

(C) $\frac{1}{3}\ln\left|\frac{2x-7}{(x-2)(x-5)}\right| + C$

(D) $\frac{1}{3}\ln\left|\frac{x-2}{x-5}\right| + C$

(E) $\frac{1}{3}\ln\left|\frac{x-5}{x-2}\right| + C$



5. The figure above shows the graph of the function f. Which of the following statements are true?

I. $\lim_{x \to 2^-} f(x) = f(2)$

II. $\lim_{x \to 6^-} f(x) = \lim_{x \to 6^+} f(x)$

III. $\lim_{x \to 6} f(x) = f(6)$

(A) II only

(B) III only

(C) I and II only

(D) II and III only

(E) I, II, and III

6. The infinite series $\sum_{k=1}^{\infty} a_k$ has nth partial sum $S_n = (-1)^{n+1}$ for $n \geq 1$. What is the sum of the series $\sum_{k=1}^{\infty} a_k$?

(A) -1

(B) 0

(C) $\frac{1}{2}$

(D) 1

(E) The series diverges.

7. Let f be the function defined by $f(x) = \begin{cases} x^2 + 2 & \text{for } x \leq 3, \\ 6x + k & \text{for } x > 3. \end{cases}$

If f is continuous at $x = 3$, what is the value of k ?

(A) -7 (B) 2 (C) 3 (D) 7 (E) There is no such value of k.

8. $\int_0^1 x\sqrt{1 + 8x^2}\, dx =$

(A) $\frac{1}{24}$ (B) $\frac{13}{12}$ (C) $\frac{9}{8}$ (D) $\frac{52}{3}$ (E) 18

9. The function f has a first derivative given by $f'(x) = x(x-3)^2(x+1)$. At what values of x does f have a relative maximum?

(A) -1 only (B) 0 only (C) -1 and 0 only (D) -1 and 3 only (E) -1, 0, and 3

10. What is the sum of the series $\sum_{n=1}^{\infty} \frac{(-2)^n}{e^{n+1}}$?

(A) $\frac{-2}{e^2 - 2e}$ (B) $\frac{-2}{e^2 + 2e}$ (C) $\frac{-2}{e+2}$ (D) $\frac{e}{e+2}$ (E) The series diverges.

$$f(x) = \begin{cases} 2x + 5 & \text{for } x < -1 \\ -x^2 + 6 & \text{for } x \geq -1 \end{cases}$$

11. If f is the function defined above, then $f'(-1)$ is

(A) -2 (B) 2 (C) 3 (D) 5 (E) nonexistent

12. Let f be the function given by $f(x) = 9^x$. If four subintervals of equal length are used, what is the value of the right Riemann sum approximation for $\int_0^2 f(x)\,dx$?

(A) 20 (B) 40 (C) 60 (D) 80 (E) 120

13. A rectangular area is to be enclosed by a wall on one side and fencing on the other three sides. If 18 meters of fencing are used, what is the maximum area that can be enclosed?

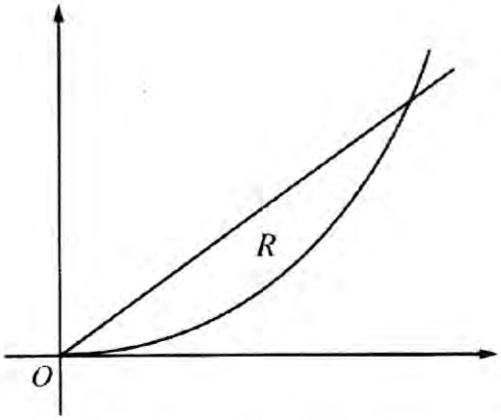
(A) $\frac{9}{2}$ m^2 (B) $\frac{81}{4}$ m^2 (C) 27 m^2 (D) 40 m^2 (E) $\frac{81}{2}$ m^2

14. Let $P(x) = 3 - 3x^2 + 6x^4$ be the fourth-degree Taylor polynomial for the function f about $x = 0$. What is the value of $f^{(4)}(0)$?

(A) 0 (B) $\frac{1}{4}$ (C) 6 (D) 24 (E) 144

15. Suppose $\ln x - \ln y = y - 4$, where y is a differentiable function of x and $y = 4$ when $x = 4$. What is the value of $\frac{dy}{dx}$ when $x = 4$?

(A) 0 (B) $\frac{1}{5}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$ (E) $\frac{17}{5}$



16. Let R be the region in the first quadrant that is bounded by the polar curves $r = \theta$ and $\theta = k$, where k is a constant, $0 < k < \frac{\pi}{2}$, as shown in the figure above. What is the area of R in terms of k ?

(A) $\frac{k^3}{6}$ (B) $\frac{k^3}{3}$ (C) $\frac{k^3}{2}$ (D) $\frac{k^2}{4}$ (E) $\frac{k^2}{2}$

17. Which of the following is the Maclaurin series for e^{3x} ?

(A) $1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$

(B) $3 + 9x + \frac{27x^2}{2} + \frac{81x^3}{3!} + \frac{243x^4}{4!} + \cdots$

(C) $1 - 3x + \frac{9x^2}{2} - \frac{27x^3}{3!} + \frac{81x^4}{4!} - \cdots$

(D) $1 + 3x + \frac{3x^2}{2} + \frac{3x^3}{3!} + \frac{3x^4}{4!} + \cdots$

(E) $1 + 3x + \frac{9x^2}{2} + \frac{27x^3}{3!} + \frac{81x^4}{4!} + \cdots$

18. $\displaystyle\int_1^{\infty} \frac{x^2}{\left(x^3+2\right)^2}\,dx$ is

(A) $-\frac{1}{9}$ (B) $\frac{1}{9}$ (C) $\frac{1}{3}$ (D) 1 (E) divergent

19. For what values of x does the graph of $y = 3x^5 + 10x^4$ have a point of inflection?

(A) $x = -\frac{8}{3}$ only

(B) $x = -2$ only

(C) $x = 0$ only

(D) $x = 0$ and $x = -\frac{8}{3}$

(E) $x = 0$ and $x = -2$

20. If $f'(x) = \dfrac{(x-2)^3\left(x^2-4\right)}{16}$ and $g(x) = f\left(x^2-1\right)$, what is $g'(2)$?

(A) 2 (B) $\frac{5}{4}$ (C) $\frac{5}{8}$ (D) $\frac{5}{16}$ (E) 0

x	1	3	5	7
$f(x)$	4	6	7	5
$f'(x)$	2	1	0	-1

21. The table above gives selected values for a differentiable function f and its first derivative. Using a left Riemann sum with 3 subintervals of equal length, which of the following is an approximation of the length of the graph of f on the interval $[1, 7]$?

(A) 6 (B) 34 (C) $2\sqrt{3} + 2\sqrt{2} + 2$ (D) $2\sqrt{5} + 2\sqrt{2} + 2$ (E) $2\sqrt{5} + 4\sqrt{2} + 2$

22. What is the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n \cdot 2^n}$?

(A) $1 < x < 5$

(B) $1 \leq x < 5$

(C) $1 \leq x \leq 5$

(D) $2 < x < 4$

(E) $2 \leq x \leq 4$

23. What is the particular solution to the differential equation $\frac{dy}{dx} = xy^2$ with the initial condition $y(2) = 1$?

(A) $y = e^{\frac{x^2}{2} - 2}$

(B) $y = e^{\frac{x^2}{2}}$

(C) $y = -\frac{2}{x^2}$

(D) $y = \frac{2}{6 - x^2}$

(E) $y = \frac{6 - x^2}{2}$

24. Which of the following series converge?

I. $1 + (-1) + 1 + \cdots + (-1)^{n-1} + \cdots$

II. $1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{2n-1} + \cdots$

III. $1 + \frac{1}{3} + \frac{1}{3^2} + \cdots + \frac{1}{3^{n-1}} + \cdots$

(A) I only

(B) II only

(C) III only

(D) II and III only

(E) I, II, and III

25. What is the slope of the line tangent to the polar curve $r = \cos\theta$ at the point where $\theta = \frac{\pi}{6}$?

(A) $-\sqrt{3}$ (B) $-\frac{1}{\sqrt{3}}$ (C) $\frac{1}{\sqrt{3}}$ (D) $\frac{\sqrt{3}}{2}$ (E) $\sqrt{3}$

26. For $x > 0$, $\frac{d}{dx}\int_{1}^{\sqrt{x}} \frac{1}{1+t^2}\,dt =$

(A) $\frac{1}{2\sqrt{x}(1+x)}$ (B) $\frac{1}{2\sqrt{x}\left(1+\sqrt{x}\right)}$ (C) $\frac{1}{1+x}$ (D) $\frac{\sqrt{x}}{1+x}$ (E) $\frac{1}{1+\sqrt{x}}$

27. What is the coefficient of x^2 in the Taylor series for $\sin^2 x$ about $x = 0$?

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

28. The function h is given by $h(x) = x^5 + 3x - 2$ and $h(1) = 2$. If h^{-1} is the inverse of h, what is the value of $\left(h^{-1}\right)'(2)$?

(A) $\frac{1}{83}$ (B) $\frac{1}{8}$ (C) $\frac{1}{2}$ (D) 1 (E) 8

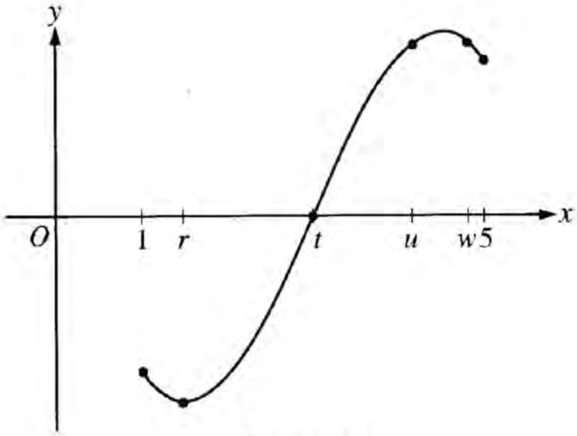
END OF PART A OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY
CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS BC
SECTION I, Part B
Time—50 minutes
Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.



Graph of f

29. The figure above shows the graph of the differentiable function f for $1 \le x \le 5$. Which of the following could be the x-coordinate of a point at which the line tangent to the graph of f is parallel to the secant line through the points $(1, f(1))$ and $(5, f(5))$?

(A) r (B) t (C) u (D) w (E) There is no such point.

30. The number of antibodies y in a patient's bloodstream at time t is increasing according to a logistic differential equation. Which of the following could be the differential equation?

(A) $\frac{dy}{dt} = 0.025t$

(B) $\frac{dy}{dt} = 0.025t(5000 - t)$

(C) $\frac{dy}{dt} = 0.025y$

(D) $\frac{dy}{dt} = 0.025(5000 - y)$

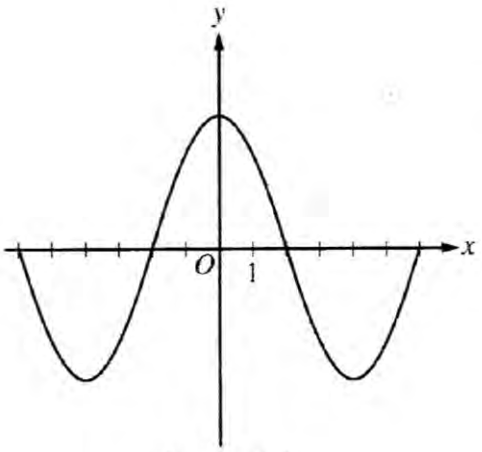
(E) $\frac{dy}{dt} = 0.025y(5000 - y)$

31. What is the area of the region enclosed by the graphs of $y = \frac{1}{1+x^2}$ and $y = x^2 - \frac{1}{3}$?

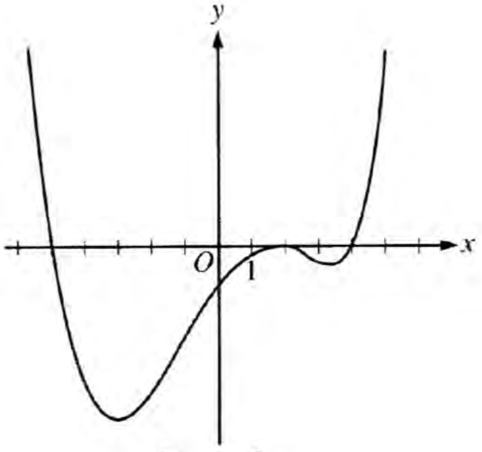
(A) 0.786 (B) 0.791 (C) 1.582 (D) 1.837 (E) 1.862

32. A vase has the shape obtained by revolving the curve $y = 2 + \sin x$ from $x = 0$ to $x = 5$ about the x-axis, where x and y are measured in inches. What is the volume, in cubic inches, of the vase?

(A) 10.716 (B) 25.501 (C) 33.666 (D) 71.113 (E) 80.115



Graph of f



Graph of g

33. The graphs of two differentiable functions f and g are shown above. Given $p(x) = f(x)g(x)$, which of the following statements about $p'(-2)$ is true?

(A) $p'(-2) < 0$

(B) $p'(-2) = 0$

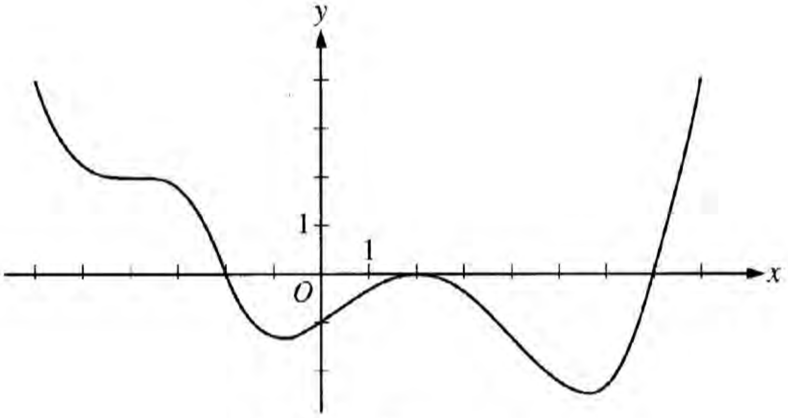
(C) $p'(-2) > 0$

(D) $p'(-2)$ is undefined.

(E) There is not enough information given to conclude anything about $p'(-2)$.

34. At time $t = 0$ years, a forest preserve has a population of 1500 deer. If the rate of growth of the population is modeled by $R(t) = 2000e^{0.23t}$ deer per year, what is the population at time $t = 3$?

(A) 3987 (B) 5487 (C) 8641 (D) 10,141 (E) 12,628



Graph of f'

35. The figure above shows the graph of f', the derivative of function f, for $-6 < x < 8$. Of the following, which best describes the graph of f on the same interval?

(A) 1 relative minimum, 1 relative maximum, and 3 points of inflection

(B) 1 relative minimum, 1 relative maximum, and 4 points of inflection

(C) 2 relative minima, 1 relative maximum, and 2 points of inflection

(D) 2 relative minima, 1 relative maximum, and 4 points of inflection

(E) 2 relative minima, 2 relative maxima, and 3 points of inflection

x	$f'(x)$
1	0.2
1.5	0.5
2	0.9

36. The table above gives values of f', the derivative of a function f . If $f(1) = 4$, what is the approximation to $f(2)$ obtained by using Euler's method with a step size of 0.5 ?

(A) 2.35

(B) 3.65

(C) 4.35

(D) 4.70

(E) 4.80

37. A sphere is expanding in such a way that the area of any circular cross section through the sphere's center is increasing at a constant rate of $2\text{cm}^2/\text{sec}$. At the instant when the radius of the sphere is 4 centimeters, what is the rate of change of the sphere's volume? (The volume V of a sphere with radius r is given by $V = \frac{4}{3}\pi r^3$.)

(A) $8\text{cm}^3/\text{sec}$

(B) $16\text{cm}^3/\text{sec}$

(C) $8\pi\,\text{cm}^3/\text{sec}$

(D) $64\pi\,\text{cm}^3/\text{sec}$

(E) $128\pi\,\text{cm}^3/\text{sec}$

38. For $t \geq 0$, the components of the velocity of a particle moving in the xy-plane are given by the parametric equations $x'(t) = \frac{1}{t+1}$ and $y'(t) = ke^{kt}$, where k is a positive constant. The line $y = 4x + 3$ is parallel to the line tangent to the path of the particle at the point where $t = 2$. What is the value of k?

(A) 0.072 (B) 0.433 (C) 0.495 (D) 0.803 (E) 0.828

t (hours)	0	1	2	3	4	5	6
$s(t)$ (miles)	0	25	55	92	150	210	275

39. The table above gives the distance $s(t)$, in miles, that a car has traveled at various times t, in hours, during a 6-hour trip. The graph of the function s is increasing and concave up. Based on the information, which of the following could be the velocity of the car, in miles per hour, at time $t = 3$?

(A) 37 (B) 49 (C) 58 (D) 65 (E) 92

40. If $0 < b_n < a_n$ for $n \geq 1$, which of the following must be true?

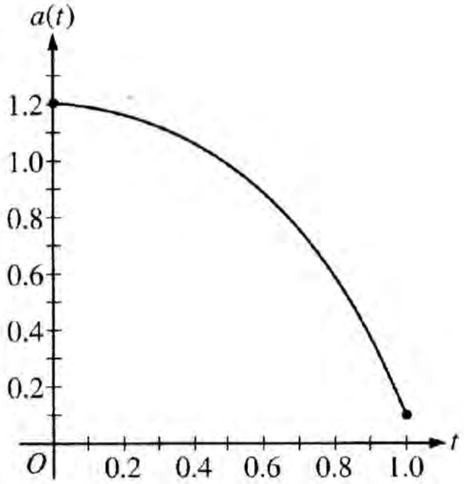
(A) If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} b_n$ converges.

(B) If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} b_n = 0$.

(C) If $\sum_{n=1}^{\infty} b_n$ diverges, then $\lim_{n\to\infty} a_n = 0$.

(D) If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges.

(E) If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.



41. A particle moves along the x-axis so that its acceleration $a(t)$ is given by the graph above for all values of t where $0 \le t \le 1$. At time $t = 0$, the velocity of the particle is $-\frac{1}{2}$. Which of the following statements must be true?

(A) The particle passes through $x = 0$ for some t between $t = 0$ and $t = 1$.

(B) The velocity of the particle is 0 for some t between $t = 0$ and $t = 1$.

(C) The velocity of the particle is negative for all values of t between $t = 0$ and $t = 1$.

(D) The velocity of the particle is positive for all values of t between $t = 0$ and $t = 1$.

(E) The velocity of the particle is less than $-\frac{1}{2}$ for all values of t between $t = 0$ and $t = 1$.

42. The function f is given by $f(x) = \int_1^x \sqrt{t^3 + 2}\,dt$. What is the average rate of change of f over the interval $[0, 3]$?

(A) 1.324 (B) 1.497 (C) 1.696 (D) 2.266 (E) 2.694

43. A particle moves along a line so that its velocity is given by $v(t) = -t^3 + 2t^2 + 2^{-t}$ for $t \ge 0$. For what values of t is the speed of the particle increasing?

(A) $(0, 0.177)$ and $(1.256, \infty)$

(B) $(0, 1.256)$ only

(C) $(0, 2.057)$ only

(D) $(0.177, 1.256)$ only

(E) $(0.177, 1.256)$ and $(2.057, \infty)$

44. Line ℓ is tangent to the graph of $y = \cos x$ at the point $(k, \cos k)$, where $0 < k < \pi$. For what value of k does line ℓ pass through the origin?

(A) 0.860

(B) 1.571

(C) 2.356

(D) 2.798

(E) There is no such value of k.

x	2	4
$f(x)$	7	13
$g(x)$	2	9
$g'(x)$	1	7
$g''(x)$	5	8

45. The table above gives selected values of twice-differentiable functions f and g, as well as the first two derivatives of g. If $f'(x) = 3$ for all values of x, what is the value of $\int_2^4 f(x)g''(x)\,dx$?

(A) 63 (B) 69 (C) 78 (D) 84 (E) 103

Name ___________________

Periods _____

CALCULUS BC
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

t (hours)	0	0.4	0.8	1.2	1.6	2.0	2.4
$v(t)$ (miles per hour)	0	11.8	9.5	17.2	16.3	16.8	20.1

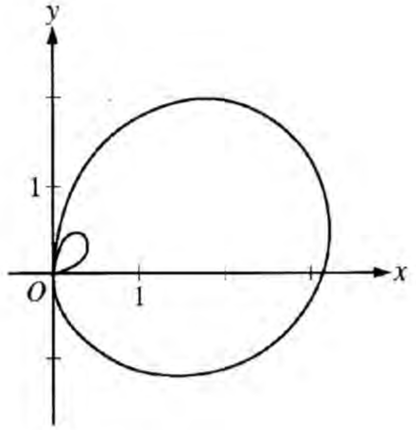
1. Ruth rode her bicycle on a straight trail. She recorded her velocity $v(t)$, in miles per hour, for selected values of t over the interval $0 \le t \le 2.4$ hours, as shown in the table above. For $0 < t \le 2.4$, $v(t) > 0$.

(a) Use the data in the table to approximate Ruth's acceleration at time $t = 1.4$ hours. Show the computations that lead to your answer. Indicate units of measure.

(b) Using correct units, interpret the meaning of $\int_0^{2.4} v(t)\,dt$ in the context of the problem. Approximate $\int_0^{2.4} v(t)\,dt$ using a midpoint Riemann sum with three subintervals of equal length and values from the table.

(c) For $0 \le t \le 2.4$ hours, Ruth's velocity can be modeled by the function g given by $g(t) = \dfrac{24t + 5\sin(6t)}{t + 0.7}$. According to the model, what was Ruth's average velocity during the time interval $0 \le t \le 2.4$?

(d) According to the model given in part (c), is Ruth's speed increasing or decreasing at time $t = 1.3$? Give a reason for your answer.



2. Consider the polar curve defined by the function $r(\theta) = \theta\cos\theta$, where $0 \le \theta \le \frac{3\pi}{2}$. The derivative of r is given by $\frac{dr}{d\theta} = \cos\theta - \theta\sin\theta$. The figure above shows the graph of r for $0 \le \theta \le \frac{3\pi}{2}$.

(a) Find the area of the region enclosed by the inner loop of the curve.

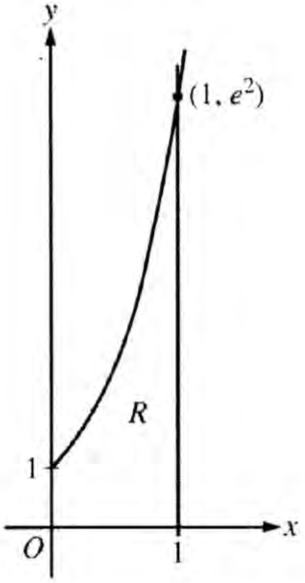
(b) For $0 \le \theta \le \frac{3\pi}{2}$, find the greatest distance from any point on the graph of r to the origin. Justify your answer.

(c) There is a point on the curve at which the slope of the line tangent to the curve is $\frac{2}{2-\pi}$. At this point, $\frac{dy}{d\theta} = \frac{1}{2}$. Find $\frac{dx}{d\theta}$ at this point.

Do not go on to Part B until you are told to do so. You will be able to work more on these first two problems but you will not be allowed to use your calculator.

CALCULUS BC
SECTION II, Part B
No calculator is allowed for these problems.

Time—60 minutes
Number of problems—4

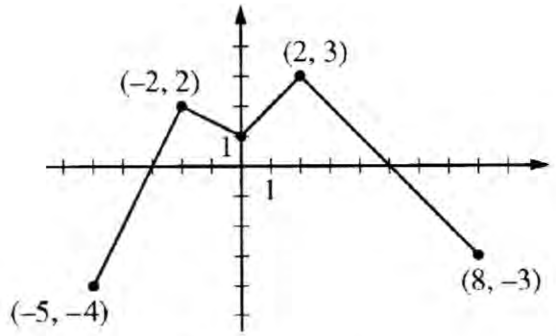


3. Let $f(x) = e^{2x}$. Let R be the region in the first quadrant bounded by the graph of $y = f(x)$ and the vertical line $x = 1$, as shown in the figure above.

(a) Write an equation for the line tangent to the graph of f at $x = 1$.

(b) Find the area of R.

(c) Region R forms the base of a solid whose cross sections perpendicular to the y-axis are squares. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.



Graph of f

4. The continuous function f is defined on the interval $-5 \le x \le 8$. The graph of f, which consists of four line segments, is shown in the figure above. Let g be the function given by $g(x) = 2x + \int_{-2}^{x} f(t)\, dt$.

(a) Find $g(0)$ and $g(-5)$.

(b) Find $g'(x)$ in terms of $f(x)$. For each of $g''(4)$ and $g''(-2)$, find the value or state that it does not exist.

(c) On what intervals, if any, is the graph of g concave down? Give a reason for your answer.

(d) The function h is given by $h(x) = g\left(x^3 + 1\right)$. Find $h'(1)$. Show the work that leads to your answer.

5. A toy train moves along a straight track set up on a table. The position $x(t)$ of the train at time t seconds is measured in centimeters from the center of the track. At time $t = 1$, the train is 6 centimeters to the left of the center, so $x(1) = -6$. For $0 \le t \le 4$, the velocity of the train at time t is given by $v(t) = 3t^2 - 12$, where $v(t)$ is measured in centimeters per second.

(a) For $0 \le t \le 4$, find $x(t)$.

(b) Find the total distance traveled by the train during the time interval $0 \le t \le 4$.

(c) A toy bus moving on the same table has position given by $(x(t), y(t))$. Here, $x(t)$ is the function found in part (a), and $y(t) = 2 + 12\sin\left(\frac{t}{4}\right)$ is the distance from the bus to the train track, in centimeters. Write, but do not evaluate, an integral expression that gives the total distance traveled by the bus during the time interval $0 \le t \le 4$.

6. Let $a_n = \dfrac{1}{n \ln n}$ for $n \geq 3$.

(a) Let f be the function given by $f(x) = \dfrac{1}{x \ln x}$. For $x \geq 3$, f is continuous, decreasing, and positive. Use the integral test to show that $\sum_{n=3}^{\infty} a_n$ diverges.

(b) Consider the infinite series $\sum_{n=3}^{\infty} (-1)^{n+1} a_n = \dfrac{1}{3 \ln 3} - \dfrac{1}{4 \ln 4} + \dfrac{1}{5 \ln 5} - \cdots$. Identify properties of this series that guarantee the series converges. Explain why the sum of this series is less than $\dfrac{1}{3}$.

(c) Find the interval of convergence of the power series $\sum_{n=3}^{\infty} \dfrac{(x-2)^{n+1}}{n \ln n}$. Show the analysis that leads to your answer.

Answer Key for AP Calculus BC
Practice Exam, Section I

Question 1: C

Question 2: B

Question 3: D

Question 4: E

Question 5: C

Question 6: E

Question 7: A

Question 8: B

Question 9: A

Question 10: B

Question 11: E

Question 12: C

Question 13: E

Question 14: E

Question 15: B

Question 16: A

Question 17: E

Question 18: B

Question 19: B

Question 20: B

Question 21: D

Question 22: B

Question 23: D

Question 24: C

Question 25: B

Question 26: A

Question 27: D

Question 28: B

Question 76: C 29.

Question 77: E 30.

Question 78: C 31.

Question 79: E 32.

Question 80: A 33.

Question 81: D 34.

Question 82: A 35.

Question 83: C 36.

Question 84: B 37.

Question 85: C 38.

Question 86: B 39.

Question 87: B 40.

Question 88: B 41.

Question 89: E 42.

Question 90: E 43.

Question 91: D 44.

Question 92: A 45.

2014 BC Mult. Choice Solutions

1. $\int (x + 5x^{-2})\, dx$

$\frac{1}{2}x^2 - 5x^{-1} + C$

$\boxed{C}$

2. $y' = \frac{2}{2x}$

$= \frac{1}{x}$

$y'\big|_{x=4} = \frac{1}{4}$

$\boxed{B}$

3. $\lim\limits_{x \to 0} \frac{2x}{\sin x}$

$\lim\limits_{x \to 0} \frac{2}{\cos x}$

2

$\boxed{D}$

4. $\int \frac{1}{(x-5)(x-2)}\, dx$

$\frac{A}{x-5} + \frac{B}{x-2} = \frac{1}{(x-5)(x-2)}$

$A(x-2) + B(x-5) = 1$

$x = 2 \rightarrow -3B = 1$
$B = -\frac{1}{3}$

$x = 5 \rightarrow 3A = 1$
$A = \frac{1}{3}$

$\frac{1}{3}\int \frac{1}{x-5}\, dx - \frac{1}{3}\int \frac{1}{x-2}\, dx$

$\frac{1}{3}\ln|x-5| - \frac{1}{3}\ln|x-2| + C$

$\frac{1}{3}\ln\left|\frac{x-5}{x-2}\right| + C$

$\boxed{E}$

5. I. $\lim\limits_{x \to 2^-} f(x) = -1$
$f(2) = -1$ True

II. $\lim\limits_{x \to 6^-} f(x) = -1$
$\lim\limits_{x \to 6^+} f(x) = -1$ True

III. $\lim\limits_{x \to 6} f(x) = -1$
$f(6) = -3$ False

$\boxed{C}$

6. If $S_n = (-1)^{n+1}$,

$\sum\limits_{K=1}^{\infty} a_K = 1 - 2 + 2 - 2 + \ldots$

$\lim\limits_{K \to \infty} a_K \neq 0$

the series div. by n^{th} term test

$\boxed{E}$

7. $3^2 + 2 = 6 \cdot 3 + k$

$11 = 18 + k$

$-7 = k$

$\boxed{A}$

8. $\frac{1}{16}\int_0^1 16x\,(1+8x^2)^{\frac{1}{2}}\, dx$

$\frac{1}{16} \cdot \frac{2}{3}\left(1+8x^2\right)^{\frac{3}{2}}\Big|_0^1$

$\frac{1}{24} \cdot 9^{\frac{3}{2}} - \frac{1}{24}$

$\frac{27}{24} - \frac{1}{24}$

$\frac{13}{12}$

$\boxed{B}$

9. CN $x = 0, 3, -1$

rel. max. @ $x = -1$

$\boxed{A}$

2014 BC Mult. Choice Solutions (continued)

10. $\sum_{n=1}^{\infty} \frac{(-2)^n}{e^{n+1}} = \frac{-2}{e^2} + \frac{4}{e^3} - \cdots$

$\text{Sum} = \frac{\frac{-2}{e^2}}{1 - \frac{-2}{e}} \cdot \frac{e^2}{e^2}$

$= \frac{-2}{e^2+2e}$

$\boxed{B}$

11. $\lim_{x \to -1^-} f(x) = 2(-1)+5$

$= 3$

$\lim_{x \to -1^+} f(x) = -(-1)^2+6$

$= 5$

f is not continuous @ $x = -1$

f is also not differentiable @ $x = -1$

$\boxed{E}$

12. $\frac{1}{2}\left(9^{\frac{1}{2}} + 9^1 + 9^{\frac{3}{2}} + 9^2\right)$

$\frac{1}{2}(3+9+27+81)$

60

$\boxed{C}$

13.

$2x+y=18$

$y = 18-2x$

$A = xy$

$A = x(18-2x)$

$A = 18x-2x^2$

$A' = 18-4x$

$0 = 18-4x$

$\frac{9}{2} = x$

$A = 18 \cdot \frac{9}{2} - 2\left(\frac{9}{2}\right)^2$

$= 81 - \frac{81}{2}$

$= 81/2$

$\boxed{E}$

14. $\frac{f^{(4)}(0)}{4!} = 6$

$f^{(4)}(0) = 6 \cdot 4!$

$= 144$

$\boxed{E}$

15. $\frac{1}{x} - \frac{y'}{y} = y'$

$\frac{1}{4} - \frac{y'}{4} = y'$ @ $(4,4)$

$\frac{1}{4} = \frac{5}{4}y'$

$\frac{1}{5} = y'$

$\boxed{B}$

16. $A = \frac{1}{2}\int_0^k \theta^2 \, d\theta$

$= \frac{1}{2} \cdot \frac{1}{3}\theta^3 \Big|_0^k$

$= \frac{1}{6}k^3$

$\boxed{A}$

17. $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots$

$e^{3x} = 1 + 3x + \frac{(3x)^2}{2} + \frac{(3x)^3}{3!} + \cdots$

$= 1 + 3x + \frac{9x^2}{2} + \frac{27x^3}{3!} + \cdots$

$\boxed{E}$

18. $\lim_{b \to \infty} \frac{1}{3}\int_1^b 3x^2 (x^3+2)^{-2} dx$

$\lim_{b \to \infty} -\frac{1}{3}(x^3+2)^{-1} \Big|_1^b$

$\lim_{b \to \infty} \frac{-1}{3(b^3+2)} + \frac{1}{3} \cdot \frac{1}{3}$

$0 + \frac{1}{9}$ $\boxed{B}$

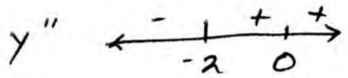
2014 BC Mult. Choice Solutions (continued)

19. $y' = 15x^4 + 40x^3$

$y'' = 60x^3 + 120x^2$

$0 = 60x^2(x+2)$

$x = 0, -2$



$\boxed{B}$

20. $g(x) = f(x^2-1)$

$g'(x) = f'(x^2-1) \cdot 2x$

$= \dfrac{(x^2-1-2)^3((x^2-1)^2-4)}{16} \cdot 2x$

$g'(2) = \dfrac{(4-1-2)^3((4-1)^2-4)}{16} \cdot 4$

$= \dfrac{5}{16} \cdot 4$

$= \dfrac{5}{4}$ $\boxed{B}$

21. Arc Length $= \int_1^7 \sqrt{1+(f'(x))^2}\,dx$

$AL \approx 2\left(\sqrt{1+(f'(1))^2} + \sqrt{1+(f'(3))^2} + \sqrt{1+(f'(5))^2}\right)$

$= 2(\sqrt{5} + \sqrt{2} + \sqrt{1})$

$= 2\sqrt{5} + 2\sqrt{2} + 2$

$\boxed{D}$

22. $\lim\limits_{n\to\infty} \left| \dfrac{(x-3)^{n+1}}{(n+1)2^{n+1}} \cdot \dfrac{n \cdot 2^n}{(x-3)^n} \right|$

$\left|\dfrac{x-3}{2}\right| < 1$

$-1 < \dfrac{x-3}{2} < 1$

$-2 < x-3 < 2$

$1 < x < 5$

check $x=1$	check $x=5$
$\sum \dfrac{(-2)^n}{n \cdot 2^n}$	$\sum \dfrac{2^n}{n \cdot 2^n}$
$\sum \dfrac{(-1)^n}{n}$	$\sum \dfrac{1}{n}$
alt. harmonic converges	harmonic diverges

IOC: $1 \le x < 5$

$\boxed{B}$

23. $\frac{1}{y^2}\,dy = x\,dx$

$\int y^{-2}\,dy = \int x\,dx$

$-\frac{1}{y} = \frac{1}{2}x^2 + C$

$y(2) = 1 \rightarrow -1 = 2 + C$

$-3 = C$

$-\frac{1}{y} = \frac{1}{2}x^2 - 3$

$\frac{1}{y} = -\frac{1}{2}x^2 + 3$

$y = \dfrac{1}{-\frac{1}{2}x^2 + 3} \cdot \dfrac{2}{2}$

$y = \dfrac{2}{-x^2+6}$ $\boxed{D}$

2014 BC Mult. Choice Solutions (continued)

24. I. div. by nTT

II. compare to $\Sigma \frac{1}{n}$

harmonic (div.)

$$\lim_{n\to\infty} \frac{1}{2n-1} \cdot \frac{n}{1} = \frac{1}{2}$$

$\Sigma \frac{1}{2n-1}$ also diverges by LCT

III G.S. w/ $r = \frac{1}{3} < 1$

conv. by GST

$\boxed{C}$

25. $y = r\sin\theta$

$y = \cos\theta\sin\theta$

$$\frac{dy}{d\theta} = \cos\theta\cos\theta + \sin\theta(-\sin\theta)$$

$$= \cos^2\theta - \sin^2\theta$$

$x = r\cos\theta$

$x = (\cos\theta)^2$

$$\frac{dx}{d\theta} = 2\cos\theta\,(-\sin\theta)$$

$$\frac{dy}{dx} = \frac{\cos^2\theta - \sin^2\theta}{-2\cos\theta\sin\theta}$$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{6}} = \frac{\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2}{-2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)}$$

$$= \frac{\frac{3}{4} - \frac{1}{4}}{-\frac{\sqrt{3}}{2}}$$

$$= \frac{1}{2} \cdot -\frac{2}{\sqrt{3}}$$

$$= -\frac{1}{\sqrt{3}}$$

$\boxed{B}$

26. $\frac{1}{1+(\sqrt{x})^2} \cdot \frac{1}{2}x^{-\frac{1}{2}}$

$$\frac{1}{1+x} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{x}}$$

$$\frac{1}{2\sqrt{x}\,(1+x)}$$

$\boxed{A}$

27. $f(x) = (\sin x)^2$

$f'(x) = 2\sin x\cos x$

$f''(x) = 2\sin x(-\sin x) + \cos x \cdot 2\cos x$

$= -2\sin^2 x + 2\cos^2 x$

$f''(0) = 2$

coeff of $x^2 = \frac{2}{2!}$

$= 1$

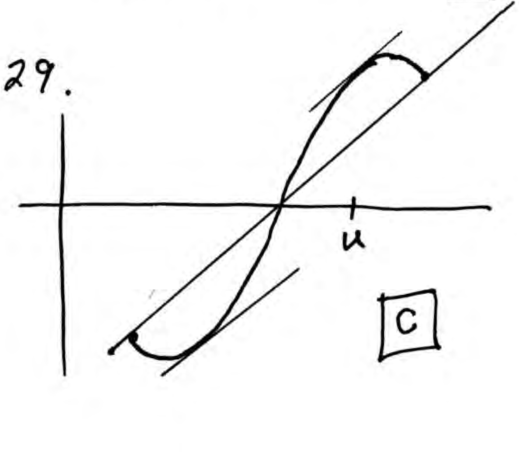
$\boxed{D}$

28. $h'(x) = 5x^4 + 3$

$h'(1) = 8$

$(h^{-1})'(2) = \frac{1}{8}$ $\boxed{B}$

29.



$\boxed{C}$

2014 BC Mult. Choice Solutions (continued)

38. $\frac{dy}{dx} = 4$ @ $t = 2$

$$\frac{ke^{kt}}{\frac{1}{t+1}} = 4 \text{ @ } t = 2$$

$$\frac{ke^{2k}}{\frac{1}{3}} = 4$$

$$ke^{2k} = \frac{4}{3}$$

$$k = .495 \qquad \boxed{C}$$

39. If s is inc. and ccu,
v is pos. and inc.

on $[2, 3]$ $V_{avg} = \frac{92-55}{3-2} = 37$

on $[3, 4]$ $V_{avg} = \frac{150-92}{4-3} = 58$

$$37 < V(3) < 58 \qquad \boxed{B}$$

40.

(A) $\lim_{n \to \infty} b_n = 0$ but the series may not converge

(B) $\lim_{n \to \infty} a_n = 0$ since $\sum a_n$ conv.

$\lim_{n \to \infty} b_n = 0$ <u>True</u>

(C) $\sum a_n$ also div, but still may approach 0

(D) $\sum b_n$ may conv.

(E) $\sum a_n$ may div. $\boxed{B}$

41. $v(t) = -\frac{1}{2} + \int_0^t a(x)\,dx$

for $0 \le t \le 1$

$0 \le \int_0^t a(x)dx <$ about 1

for $0 \le t \le 1$

$-\frac{1}{2} \le v(t) <$ about $\frac{1}{2}$

$\boxed{B}$

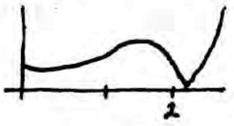
42. $$\text{AROC} = \frac{\int_1^3 \sqrt{t^3+2}\,dt - \int_1^0 \sqrt{t^3+2}\,dt}{3-0}$$

$$= 2.694$$

$\boxed{E}$

43. <u>Method 1</u>

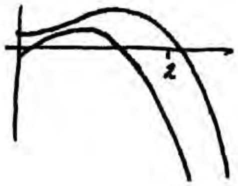
graph speed $= |-t^3+2t^2+2^{-t}|$
and look for positive slopes



$\boxed{E}$

<u>Method 2</u>

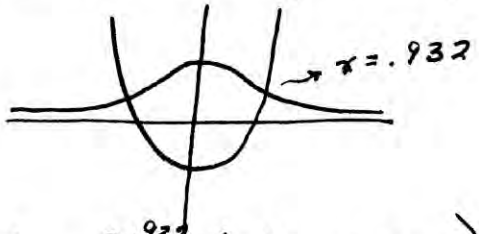
graph $v(t)$ and its derivative and look for the same sign



2014 BC Mult. Choice Solutions (continued)

30. only $\boxed{E}$ is the form of a logistic differential equation

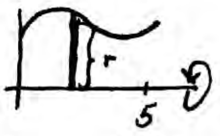
31.



$$A = \int_{-.932}^{.932}\left(\frac{1}{1+x^2} - \left(x^2 - \frac{1}{3}\right)\right)dx$$

$= 1.582$ $\boxed{C}$

32.



disc method

$$V = \pi\int_0^5 (2+\sin x)^2\,dx$$

$= 80.115$ $\boxed{E}$

33. $p'(x) = f(x)g'(x) + g(x)f'(x)$

$p'(-2) = f(-2)g'(-2) + g(-2)f'(-2)$

$= 0 \cdot g'(-2) + (\text{neg.})(\text{pos.})$

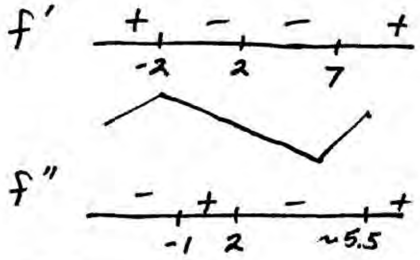
$= \text{neg.}$ $\boxed{A}$

34. $Pop(3) = 1500 + \int_0^3 2000e^{.23t}\,dt$

$= 10141$

$\boxed{D}$

35.



3 POI

$\boxed{A}$

36.

x	y	$\frac{dy}{dx}$	$\Delta y = m\,\Delta x$
1	4	.2	$\Delta y = .2(.5) = .1$
1.5	4.1	.5	$\Delta y = .5(.5) = .25$
2	4.35		

$\boxed{C}$

37. $A = \pi r^2$

$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$

when $r = 4$

$2 = 2\pi \cdot 4\frac{dr}{dt}$

$\frac{1}{4\pi} = \frac{dr}{dt}$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$

$= 4\pi \cdot 16 \cdot \frac{1}{4\pi}$

$= 16$ $\boxed{B}$

2014 BC Mult. Choice Solutions (continued)

44. $y' = -\sin x$

$y'\big|_{x=k} = -\sin k$

Tan. Line:

$y - \cos k = -\sin k\,(x-k)$

if T.L. passes through (0,0)

$-\cos k = -\sin k\,(-k)$

on [.860, 2.798]

$k = 2.798$ $\boxed{D}$

45. $u = f(x)$

$du = f'(x)\,dx$

$dv = g''(x)\,dx$

$v = g'(x)$

$$\int_2^4 f(x)\,g''(x)\,dx = \left(f(x)\,g'(x) - \int g'(x)\,f'(x)\,dx\right)\Big|_2^4$$

$$= \left(f(x)\,g'(x) - 3\int g'(x)\,dx\right)\Big|_2^4$$

$$= \left(f(x)g'(x) - 3\,g(x)\right)\Big|_2^4$$

$$= \left(f(4)\,g'(4) - 3\,g(4)\right) - \left(f(2)\,g'(2) - 3g(2)\right)$$

$$= (13\cdot 7 - 3\cdot 9) - (7\cdot 1 - 3\cdot 2)$$

$$= 63 \qquad \boxed{A}$$

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Question 1

t (hours)	0	0.4	0.8	1.2	1.6	2.0	2.4
$v(t)$ (miles per hour)	0	11.8	9.5	17.2	16.3	16.8	20.1

Ruth rode her bicycle on a straight trail. She recorded her velocity $v(t)$, in miles per hour, for selected values of t over the interval $0 \le t \le 2.4$ hours, as shown in the table above. For $0 < t \le 2.4$, $v(t) > 0$.

(a) Use the data in the table to approximate Ruth's acceleration at time $t = 1.4$ hours. Show the computations that lead to your answer. Indicate units of measure.

(b) Using correct units, interpret the meaning of $\int_0^{2.4} v(t)\,dt$ in the context of the problem. Approximate $\int_0^{2.4} v(t)\,dt$ using a midpoint Riemann sum with three subintervals of equal length and values from the table.

(c) For $0 \le t \le 2.4$ hours, Ruth's velocity can be modeled by the function g given by $g(t) = \dfrac{24t + 5\sin(6t)}{t + 0.7}$. According to the model, what was Ruth's average velocity during the time interval $0 \le t \le 2.4$?

(d) According to the model given in part (c), is Ruth's speed increasing or decreasing at time $t = 1.3$? Give a reason for your answer.

(a) $a(1.4) \approx \dfrac{v(1.6) - v(1.2)}{1.6 - 1.2} = \dfrac{16.3 - 17.2}{1.6 - 1.2} = -2.25 \text{ miles/hr}^2$

$2 : \begin{cases} 1 : \text{approximation} \\ 1 : \text{units} \end{cases}$

(b) $\int_0^{2.4} v(t)\,dt$ is the total distance Ruth traveled, in miles, from time $t = 0$ to time $t = 2.4$ hours.

$$\int_0^{2.4} v(t)\,dt \approx (0.8)(11.8) + (0.8)(17.2) + (0.8)(16.8) = 36.64 \text{ miles}$$

$3 : \begin{cases} 1 : \text{interpretation} \\ 1 : \text{midpoint Riemann sum} \\ 1 : \text{approximation} \end{cases}$

(c) Average velocity $= \dfrac{1}{2.4}\int_0^{2.4} g(t)\,dt = 14.064$ miles/hr

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(d) Velocity $= g(1.3) = 18.096358 > 0$
Acceleration $= g'(1.3) = 3.761152 > 0$

Ruth's speed is increasing at time $t = 1.3$ since velocity and acceleration have the same sign at this time.

2 : conclusion with reason

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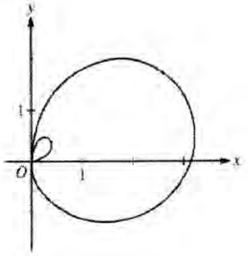
Question 2



Consider the polar curve defined by the function $r(\theta) = \theta \cos \theta$, where $0 \le \theta \le \frac{3\pi}{2}$. The derivative of r is given by $\frac{dr}{d\theta} = \cos \theta - \theta \sin \theta$. The figure above shows the graph of r for $0 \le \theta \le \frac{3\pi}{2}$.

(a) Find the area of the region enclosed by the inner loop of the curve.

(b) For $0 \le \theta \le \frac{3\pi}{2}$, find the greatest distance from any point on the graph of r to the origin. Justify your answer.

(c) There is a point on the curve at which the slope of the line tangent to the curve is $\frac{2}{2-\pi}$. At this point, $\frac{dy}{d\theta} = \frac{1}{2}$. Find $\frac{dx}{d\theta}$ at this point.

(a) Area $= \frac{1}{2}\int_0^{\pi/2} (r(\theta))^2 \, d\theta = 0.127$ (or 0.126)

$$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and constant} \\ 1 : \text{answer} \end{cases}$$

(b) $r'(\theta) = \cos \theta - \theta \sin \theta = 0 \Rightarrow \theta = 0.860334, 3.425618$

θ	$r(\theta)$
0	0
0.860334	0.561096
3.425618	−3.288371
$\frac{3\pi}{2}$	0

Therefore, the greatest distance from any point on the graph of r to the origin is 3.288.

$$3 : \begin{cases} 1 : \text{identifies } \theta = 3.425618 \text{ as a candidate} \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$$

(c) $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$

At the point where the tangent line has slope $\frac{2}{2-\pi}$,

$\frac{2}{2-\pi} = \frac{1/2}{dx/d\theta}$.

Therefore, $\frac{dx}{d\theta} = \frac{1}{2} \cdot \frac{2-\pi}{2} = \frac{2-\pi}{4}$ at this point.

$$3 : \begin{cases} 1 : \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} \\ 1 : \text{equation} \\ 1 : \text{answer} \end{cases}$$

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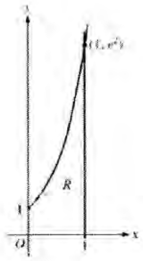
Question 3



Let $f(x) = e^{2x}$. Let R be the region in the first quadrant bounded by the graph of $y = f(x)$ and the vertical line $x = 1$, as shown in the figure above.

(a) Write an equation for the line tangent to the graph of f at $x = 1$.

(b) Find the area of R.

(c) Region R forms the base of a solid whose cross sections perpendicular to the y-axis are squares. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

(a) $f(1) = e^2$ $f'(x) = 2e^{2x} \Rightarrow f'(1) = 2e^2$ An equation for the tangent line is $y = e^2 + 2e^2(x-1)$.	$2 : \begin{cases} 1 : f'(1) \\ 1 : \text{answer} \end{cases}$
(b) Area $= \int_0^1 e^{2x}\,dx = \left[\frac{1}{2}e^{2x}\right]_{x=0}^{x=1} = \frac{1}{2}\left(e^2 - 1\right)$	$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$
(c) Volume $= 1 + \int_1^{e^2} \left(1 - \frac{1}{2}\ln y\right)^2 dy$	$4 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$

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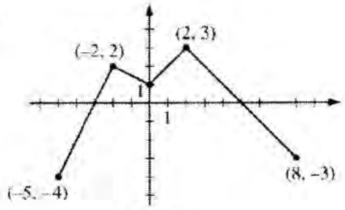
Question 4



Graph of f

The continuous function f is defined on the interval $-5 \le x \le 8$. The graph of f, which consists of four line segments, is shown in the figure above.

Let g be the function given by $g(x) = 2x + \int_{-2}^{x} f(t)\,dt$.

(a) Find $g(0)$ and $g(-5)$.

(b) Find $g'(x)$ in terms of $f(x)$. For each of $g''(4)$ and $g''(-2)$, find the value or state that it does not exist.

(c) On what intervals, if any, is the graph of g concave down? Give a reason for your answer.

(d) The function h is given by $h(x) = g\left(x^3 + 1\right)$. Find $h'(1)$. Show the work that leads to your answer.

(a) $g(0) = 2 \cdot 0 + \int_{-2}^{0} f(t)\,dt = 3$ $g(-5) = 2 \cdot (-5) + \int_{-2}^{-5} f(t)\,dt = -10 + 3 = -7$	$2 : \begin{cases} 1 : g(0) \\ 1 : g(-5) \end{cases}$
(b) $g'(x) = 2 + f(x)$ $g''(x) = f'(x)$ $g''(4) = f'(4) = -1$ $g''(-2) = f'(-2)$ does not exist.	$3 : \begin{cases} 1 : g'(x) \\ 1 : g''(4) \\ 1 : g''(-2) \text{ does not exist} \end{cases}$
(c) The graph of g is concave down on the intervals $(-2, 0)$ and $(2, 8)$ since $g'(x) = 2 + f(x)$ decreases on those intervals.	1 : intervals and reason
(d) $h'(x) = g'\left(x^3 + 1\right) \cdot 3x^2$ $h'(1) = g'(2) \cdot 3 = (2 + f(2)) \cdot 3$ $= (2 + 3) \cdot 3 = 15$	$3 : \begin{cases} 2 : \text{chain rule} \\ 1 : \text{answer} \end{cases}$

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Question 5

A toy train moves along a straight track set up on a table. The position $x(t)$ of the train at time t seconds is measured in centimeters from the center of the track. At time $t = 1$, the train is 6 centimeters to the left of the center, so $x(1) = -6$. For $0 \le t \le 4$, the velocity of the train at time t is given by $v(t) = 3t^2 - 12$, where $v(t)$ is measured in centimeters per second.

(a) For $0 \le t \le 4$, find $x(t)$.

(b) Find the total distance traveled by the train during the time interval $0 \le t \le 4$.

(c) A toy bus moving on the same table has position given by $(x(t), y(t))$. Here, $x(t)$ is the function found in part (a), and $y(t) = 2 + 12\sin\left(\frac{t}{4}\right)$ is the distance from the bus to the train track, in centimeters. Write, but do not evaluate, an integral expression that gives the total distance traveled by the bus during the time interval $0 \le t \le 4$.

(a) $$\begin{aligned} x(t) &= -6 + \int_1^t \left(3u^2 - 12\right) du \\ &= -6 + \left[u^3 - 12u\right]_{u=1}^{u=t} \\ &= t^3 - 12t + 5 \end{aligned}$$

$$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$$

(b) $v(t) = 3\left(t^2 - 4\right) = 0 \Rightarrow t = -2,\ t = 2$

$$\begin{aligned} \text{Distance} &= |x(2) - x(0)| + |x(4) - x(2)| \\ &= |-11 - 5| + |21 - (-11)| \\ &= 16 + 32 = 48 \end{aligned}$$

$$3 : \begin{cases} 1 : \text{identifies } t = 2 \\ 1 : \text{considers } x(0),\ x(2), \text{ and } x(4) \\ 1 : \text{answer} \end{cases}$$

(c) $y'(t) = 3\cos\left(\frac{t}{4}\right)$

$$\text{Distance} = \int_0^4 \sqrt{\left(3t^2 - 12\right)^2 + \left(3\cos\left(\frac{t}{4}\right)\right)^2}\, dt$$

$$3 : \begin{cases} 1 : y'(t) \\ 2 : \text{integral} \end{cases}$$

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Question 6

Let $a_n = \dfrac{1}{n \ln n}$ for $n \geq 3$.

(a) Let f be the function given by $f(x) = \dfrac{1}{x \ln x}$. For $x \geq 3$, f is continuous, decreasing, and positive.

Use the integral test to show that $\displaystyle\sum_{n=3}^{\infty} a_n$ diverges.

(b) Consider the infinite series $\displaystyle\sum_{n=3}^{\infty} (-1)^{n+1} a_n = \frac{1}{3\ln 3} - \frac{1}{4\ln 4} + \frac{1}{5\ln 5} - \cdots$. Identify properties of this series that guarantee the series converges. Explain why the sum of this series is less than $\dfrac{1}{3}$.

(c) Find the interval of convergence of the power series $\displaystyle\sum_{n=3}^{\infty} \frac{(x-2)^{n+1}}{n \ln n}$. Show the analysis that leads to your answer.

(a) $u = \ln x \Rightarrow du = \dfrac{1}{x}\,dx$

$$\int_3^{\infty} \frac{1}{x \ln x}\,dx = \lim_{b\to\infty} \int_3^b \frac{1}{x\ln x}\,dx = \lim_{b\to\infty} \int_{\ln 3}^{\ln b} \frac{1}{u}\,du = \lim_{b\to\infty} \Big[\ln|u|\Big]_{u=\ln 3}^{u=\ln b}$$
$$= \lim_{b\to\infty} \left(\ln|\ln b| - \ln|\ln 3|\right) = \infty$$

Therefore, the series diverges.

3 : { 1 : integral
1 : antiderivative
1 : evaluation

(b) The terms in this alternating series decrease in absolute value and $\displaystyle\lim_{n\to\infty} \frac{1}{n\ln n} = 0$. Therefore, the Alternating Series Test guarantees that this series converges. Furthermore,

$$\frac{1}{3\ln 3} - \frac{1}{4\ln 4} < \text{Sum} < \frac{1}{3\ln 3} < \frac{1}{3}$$

Therefore, the sum of the series is less than $\dfrac{1}{3}$.

2 : { 1 : properties
1 : explanation

(c) $$\left|\frac{(x-2)^{n+2}}{(n+1)\ln(n+1)} \cdot \frac{n\ln n}{(x-2)^{n+1}}\right| = \left(\frac{n}{n+1}\right)\left(\frac{\ln n}{\ln(n+1)}\right)|x-2|$$

$$\lim_{n\to\infty} \left(\frac{n}{n+1}\right)\left(\frac{\ln n}{\ln(n+1)}\right)|x-2| = |x-2|$$

$|x-2| < 1 \Rightarrow 1 < x < 3$

When $x = 1$, the series is $\dfrac{1}{3\ln 3} - \dfrac{1}{4\ln 4} + \dfrac{1}{5\ln 5} - \dfrac{1}{6\ln 6} + \cdots$

This series converges by the Alternating Series Test.

When $x = 3$, the series is $\dfrac{1}{3\ln 3} + \dfrac{1}{4\ln 4} + \dfrac{1}{5\ln 5} + \dfrac{1}{6\ln 6} + \cdots$

This series diverges by the integral test, as shown in part (a).
Therefore, the interval of convergence is $1 \leq x < 3$.

4 : { 1: sets up ratio
1: computes limit of ratio
1: identifies radius of convergence
1: analysis and interval of convergence

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2014 AP Calculus BC Scoring Worksheet

Section I: Multiple Choice

__________ × 1.2000 = ______________

Number Correct (out of 45)

Weighted Section I Score (Do not round)

Section II: Free Response

Question 1 __________ (out of 9) × 1.0000 = __________ (Do not round)

Question 2 __________ (out of 9) × 1.0000 = __________ (Do not round)

Question 3 __________ (out of 9) × 1.0000 = __________ (Do not round)

Question 4 __________ (out of 9) × 1.0000 = __________ (Do not round)

Question 5 __________ (out of 9) × 1.0000 = __________ (Do not round)

Question 6 __________ (out of 9) × 1.0000 = __________ (Do not round)

Sum = __________

Weighted Section II Score (Do not round)

Composite Score

__________ + __________ = __________

Weighted Section I Score

Weighted Section II Score

Composite Score (Round to nearest whole number)

AP Score Conversion Chart
Calculus BC

Composite Score Range	AP Score
62-108	5
52-61	4
40-51	3
33-39	2
0-32	1