Differentiation worksheet #1 for Calculus 1

Use the limit definition of derivative to differentiate the following functions:

(1) $x + 1$
(2) $2/x$
(3) $x^2 + x$
(4) $\sqrt{x - 4}$
(5) $\dfrac{x}{x+1}$ (without simplifying first!)

Here is a list of differentiation rules:

A. Power Rule B. Chain Rule
C. Product Rule D. Quotient Rule

For each given function, mark which differentiation rule you would apply *first*.

(6) $x^{3.2}$
(7) $x^2 - x^4$
(8) $\dfrac{\pi x}{6}$
(9) $x^2\sqrt{x^4 + 1}$
(10) $\cos(x^4)$
(11) $\dfrac{x}{1-x}$
(12) $(x + x^{1/2})(\cos(x) + x^{1/3})$
(13) $\cos^4(x)$
(14) $\cos(x^4) - 2\sin(x)$
(15) x^π

You are given the following information: f and g are continuous differentiable functions such that $f(5) = 3$, $g(5) = 4$, $f'(5) = 7$, $g'(5) = -2$. Find $h(5)$ and $h'(5)$ for each of the following functions h:

(16) $h = f + g$
(17) $h = f - g$
(18) $h = 2f/g$
(19) $h = g^2 - 3g$
(20) $h = \sqrt{f + f * g}$
(21) $h = \dfrac{f^2 + g}{f + g}$

Find the derivative. Show **every** time you use one of the rules A–D above.

Example. To find the derivative of $x^3 + \sin(x)$:

$$\begin{aligned} \frac{d}{dx}(x^3 + \sin(x)) &= \frac{d}{dx}(x^3) + \frac{d}{dx}(\sin(x)) \\ &= 3x^2 + \cos(x) \end{aligned}$$

A. power rule

(22) $x^{3.2}$
(23) $x^5 + 2x^{4.3} + \frac{1}{4}x^{1/3}$
(24) $-x^{2+1}$
(25) -5
(26) $5/x^2$
(27) $25 - \frac{x}{3}$
(28) $\sqrt[3]{x} + \sqrt[4]{x}$
(29) $1/x^{-0.4}$
(30) $\dfrac{\pi x}{6}$
(31) $x^\pi - \pi x$
(32) $x^{-7} - 7x^{-1}$
(33) $\dfrac{3}{4}x^{-4/7} + 3x^{-\pi}$
(34) $x^{\sqrt{5}} + \sqrt{5x}$
(35) $ax^2 + bx + c$, where a, b, c are constants
(36) $5\cos(x) - 2\cos(x)$

(37) $\tan(x) + \csc(x)$
(38) $5\sin(x) + x^2$
(39) $x^{1.7}\cos(x)$
(40) $\sin(x)\cos(x)$
(41) $(x^2 + \frac{1}{x})\tan(x)$
(42) $\sqrt{x}(\sin(x) + \cos(x))$
(43) $\sin(x) + \sin(x)\cos(x)$
(44) $\dfrac{x-4}{x^2+2}$
(45) $\dfrac{x}{x+\sin(x)}$
(46) $\dfrac{2x-\sqrt{x}}{3-x}$
(47) $\dfrac{x+1}{2}$
(48) $\dfrac{\sin(x)+\sin(x)\cos(x)}{x}$
(49) $(x+1)^{-3/4}$
(50) $\sin(x^4)$
(51) $\cos(x^{5.6})$
(52) $\tan(2\pi x + \frac{\pi}{2})$
(53) $\sqrt{x^2-16}$
(54) $\sqrt{16-x^2}$
(55) $\sqrt[3]{x^3-2x+1}$
(56) $\sin^4(x)$
(57) $\sqrt{(1+\frac{1}{x})}$
(58) $\sqrt{2x+1}(x^2+1)$
(59) $\dfrac{x^2}{\sqrt{x+2}} - \dfrac{\sqrt{x+2}}{x^2}$
(60) $\cos(\sqrt{x})\sqrt{\cos(x)}$
(61) $\cos(\sqrt{x^2+2})$
(62) $\sqrt{\cos(x^2+2)} + \sqrt{\cos^2(x)+2}$
(63) $\tan(\sqrt{2x+5})$
(64) $\sin\left(\dfrac{2\sqrt{x}+1}{x+1}\right)$

Find an equation for the tangent line to the given function at the given point.

(65) $y = 2x^2 + x + 1$, at $(4, 37)$
(66) $y = x^3 - x$, at $(1/3, -8/27)$
(67) $y = x^3 - x$, at $(1/2, -3/8)$
(68) $y = x^3 - x$, at $(2/3, -10/27)$
(69) $y = \sqrt{25-x^2}$, at $(-4, 3)$
(70) $y = \sin(x) - \frac{x}{2}$, at $(\pi, -\pi/2)$

Find the second derivative of each of the following functions.

(71) $x^4 + 2x^2 + 3$
(72) $\sin(x) + 1$
(73) $\sin(2x)$
(74) $\tan(x)$
(75) $\sqrt{x}$

"Cruel and unusual derivatives." These are harder; only try them if you feel all right with everything up to this point. Find the first derivatives, showing your steps:

(76) $\sqrt{x+\sqrt{x+\sqrt{x}}}$
(77) $\sin(x)\cos(x)\sqrt{x}$
(78) $\sin(x^2)\cos(x^2)\sqrt{x^2+1}$
(79) $\dfrac{2\sin(x^2)+3\cos(\sqrt{x})}{2\sin^2(x)-5\cos^2(x)}$
(80) $\dfrac{1}{1+\frac{1}{1+\frac{1}{x}}}$