**What’s My Sum?**

**Teacher Notes**

**Synopsis**

Students use paper cutting to understand how an infinite sum can result in a finite number.

**Main Standard**

MA8.0 Students are familiar with the notion of the limit of a sequence and the limit of a function as the independent variable approaches a number or infinity. They determine whether certain sequences converge or diverge.

**Materials**

Scissors

Copies, *What’s My Sum?, Part I*

Copies, *What’s My Sum? Part II*

**Teacher Directions**

Pass out *What’s My Sum?, Part I*. You may want to model the first cut for the students. Although, in some classes you may find you don’t need to model it for them.

*Group Work*

Problem #3 - student teams may need help with the fact that this sum is really never ending. They may use ellipsis (...) to show that the sum is infinite.

Discuss problems 1-6, and then depending on when you choose to do this activity, the students may or may not know how to write the notation in number 7.

*What’s My Sum?, Part II*  are also area problems, similar to what they just put together.

You may want to model part (a) for the groups. Cut a piece of square paper into fourths and this time make 3 piles. Then take the fourth sheet and cut that into fourths and distribute the pieces into the 3 piles with one left over, etc.

**What’s My Sum?, Part I**

Follow the directions below:

Take a sheet of paper, cut it into 3 equal pieces

Take two of the pieces and put them in two separate piles (pile A and pile B)

Take the third piece, cut it into 3 equal pieces.

Take two of the pieces of paper you just cut and put one into pile A and one into pile B

Continue this process until you have “nothing” left to cut.

**Suppose the area of the original paper was 1 sq. unit.**

1. Look at pile A and at pile B, how much of the area of the original paper is **just in pile A**? (Assume that you continue to cut the third piece so small that the area could be considered negligible).
2. Take Pile A and on each piece of paper, write the area that piece directly on it. Remember the area of the original paper was considered 1 sq. unit.
3. Write the area of pile A as a sum of **all** of the pieces (note you do not have to add them, just write them as a sum).
4. What does the sum in #3 get closer and closer to the more you cut up the 3rd piece of paper? (remember your answer to #1)
5. Does the summation in #3 really ever end?

But what does this sum really get close to equaling?

1. So even though the sum never ends, we say that the sum **converges** to a number. This is an example of a **convergent series**, i.e. an **infinite series** with a **finite sum**. Write this convergent series using sigma notation.

**What’s My Sum? Part II**

The following pictures illustrate an infinite series with a finite sum. Write the series and the sum. Assume that the original area is 1 square unit.

 a) b)

Series: Series:

Sum: Sum:

c)

Series:

Sum: