CALCULUS
WORKSHEET ON AVERAGE VALUE

Work the following on **notebook paper**. Use your calculator on problems 3 – 6, and give decimal answers correct to **three** decimal places.
On problems 1 and 2,
(a) Find the average value of f on the given interval.
(b) Find the value of c such that $f_{AVE} = f(c)$.

1. $f(x) = (x-3)^2, [2, 5]$

2. $f(x) = \sqrt{x}, [0, 4]$

---

3. The table below gives values of a continuous function. Use a midpoint Riemann sum with three equal subintervals to estimate the average value of f on [20, 50].

x	20	25	30	35	40	45	50
$f(x)$	42	38	31	29	35	48	60

---

4. The velocity graph of an accelerating car is shown on the right.
(a) Estimate the average velocity of the car during the first 12 seconds by using a midpoint Riemann sum with three equal subintervals.
(b) At what time was the instantaneous velocity equal to the average velocity?

---
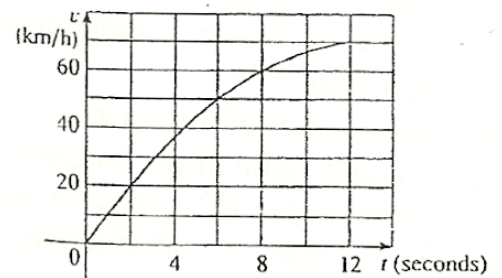
5. In a certain city, the temperature, in °F, t hours after 9 AM was modeled by the function $T(t) = 50 + 14\sin\left(\dfrac{\pi t}{12}\right)$. Find the average temperature during the period from 9 AM to 9 PM.

---

6. If a cup of coffee has temperature 95°C in a room where the temperature is 20°C, then, according to Newton's Law of Cooling, the temperature of the coffee after t minutes is given by the function $T(t) = 20 + 75e^{-t/50}$. What is the average temperature of the coffee during the first half hour?

---

7. Suppose the $C(t)$ represents the daily cost of heating your house, measured in dollars per day, where t is time measured in days and $t = 0$ corresponds to January 1, 2010.. Interpret $\int_0^{90} C(t)dt$ and $\dfrac{1}{90-0}\int_0^{90} C(t)dt$.
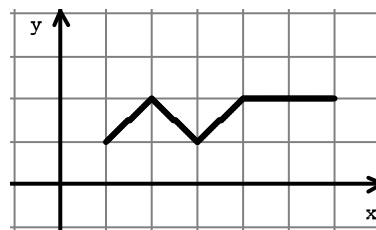
8. Using the figure on the right,

(a) Find $\int_1^6 f(x)dx$.

(b) What is the average value of f on $[1, 6]$?

Graph of f

---

9. The average value of $y = f(x)$ equals 4 for $1 \leq x \leq 6$ and equals 5 for $6 \leq x \leq 8$.

What is the average value of $f(x)$ for $1 \leq x \leq 8$?
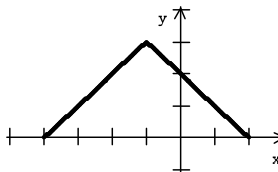
---

10. Suppose $\int_0^3 f(x)dx = 6$.

(a) What is the average value of $f(x)$ on the interval $x = 0$ to $x = 3$?

(b) If $f(x)$ is even, what is the value of $\int_{-3}^3 f(x)dx$? What is the average value of $f(x)$ on
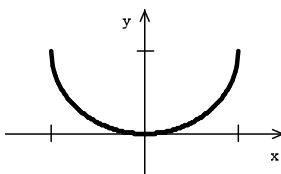
the interval $x = -3$ to $x = 3$?

(c) If $f(x)$ is odd, what is the value of $\int_{-3}^3 f(x)dx$? What is the average value of $f(x)$ on

the interval $x = -3$ to $x = 3$?

In problems 11 – 14, find the average value of the function on the given interval without integrating.
Hint: Use Geometry. (No calculator)

11. $f(x) = \begin{cases} x+4, & -4 \leq x \leq -1 \\ -x+2, & -1 \leq x \leq 2 \end{cases}$ on $[-4, 2]$

12. $f(x) = 1 - \sqrt{1-x^2}$ $[-1, 1]$

13. $f(x) = \sin x$, $[0, 2\pi]$

14. $f(x) = \tan x$, $\left[-\dfrac{\pi}{4}, \dfrac{\pi}{4}\right]$