

Related Rates

Steps:

1. Draw picture.
2. Write equation(s).
3. Write all variables (include rates of change).
4. Get equation in terms of one variable (may need another equation).
5. Take derivative with respect to t.
6. Substitute and solve.
7. Answer the question.

Right ?'s: $a^2 + b^2 = c^2$ (Ladder problems)

Circles: $A = \pi r^2$ $C = 2\pi r$

Cones: $V = \frac{1}{3}\pi r^2 h$

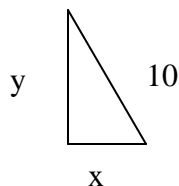
Cylinders: $V = \pi r^2 h$

Spheres: $V = \frac{4}{3}\pi r^3$ $S = 4\pi r^2$

Example: The famous ladder problem -- A.P.'s absolute favorite!

Question: A ladder, 10 feet long is leaning against a building. The bottom of the ladder is moved away from the building at the constant rate of $\frac{1}{2}$ ft/sec. Find the rate at which the ladder is falling down the building when the ladder is 6 feet from the building.

Step #1



Step #2

$$x^2 + y^2 = 10^2$$

Step #3

$x = "6"$	$\frac{dx}{dt} = \frac{1}{2}$
$y = "8" (6^2 + y^2 = 10^2)$	$\frac{dy}{dt} = ?$

Step #4

No second equation to substitute

Step #5

$$\left(\frac{dx}{dt}\right)(2x) + \left(\frac{dy}{dt}\right)(2y) = 0$$

Step #6

$$\left(\frac{1}{2}\right)(2*6) + \left(\frac{dy}{dt}\right)(2*8) = 0$$
$$6 + 16\left(\frac{dy}{dt}\right) = 0 \quad ? \quad \frac{dy}{dt} = \frac{-6}{16} = \frac{-3}{8}$$

Step #7

The ladder is falling at a rate of $\frac{3}{8}$ feet per second.

Note: You may substitute a value only **AFTER** taking the derivative if that value is changing!

The famous **CONE** problem -- A.P.'s 2nd favorite question!

If a cone is expanding or contracting, then the radius and height remain proportional $\left(\frac{r_1}{h_1} = \frac{r_2}{h_2} \right)$

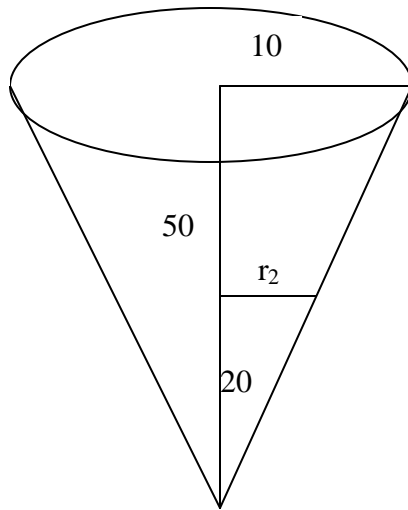
Thus, you can substitute for r such that your $V = \frac{1}{3} \mathbf{p} r^2 h$ equation will only have h .

Thus, you won't have to do the product rule!

Example:

Water is withdrawn from a conical reservoir 20 ft in diameter and 50 ft deep (vertex down) at the constant rate of $12 \frac{\text{ft}^3}{\text{min}}$. How fast is the water level falling when the depth of water in the reservoir is 20 ft?

Step #1 Draw picture of cone.



Notice the similar right triangles -- that's why the radius and height are proportional !!

Step #2 $V = \frac{1}{3} \mathbf{p} r^2 h$ $\frac{10}{50} = \frac{r_2}{h_2}$ so, $r_2 = (0.2) h_2$

Step #3

$V =$	$\frac{dV}{dt} = -12$
$r =$	$\frac{dr}{dt} =$
$h = \text{"20"}$	$\frac{dh}{dt} = ?$

Step #4 $V = \frac{1}{3} \mathbf{p} r^2 h \Rightarrow V = \frac{1}{3} \mathbf{p} (.2h)^2 h \Rightarrow V = \frac{1}{3} \mathbf{p} (.04h^2)h \Rightarrow V = \frac{\mathbf{p}}{75} h^3$

Step #5 $\frac{dV}{dt} = \frac{\mathbf{p}}{25} (h^2) \left(\frac{dh}{dt} \right)$

Step #6 $-12 = \frac{\mathbf{p}}{25} (20^2) \left(\frac{dh}{dt} \right) \Rightarrow -12 = \frac{400\mathbf{p}}{25} \left(\frac{dh}{dt} \right) \Rightarrow \frac{dh}{dt} = \frac{-3}{4\mathbf{p}}$

Step #7 The water level is falling at a rate of $\frac{-3}{4\mathbf{p}}$ feet per minute!