AP* Calculus Review

The Fundamental Theorems of Calculus

Teacher Packet

The Fundamental Theorems of Calculus

I. If f is continuous on [a, b], then the function $F(x)=\int_a^x f(t)\,dt$ has a derivative at every point in [a, b], and $\frac{dF}{dx}=\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$.

II. If f is continuous on [a, b], and if F is any antiderivative of f on [a, b], then $\int_a^b f(t)\,dt = F(b)-F(a)$.

Note: These two theorems may be presented in reverse order. Part II is sometimes called the Integral Evaluation Theorem.

Don't overlook the obvious!

1. $\frac{d}{dx}\int_a^a f(t)\,dt = 0$, because the definite integral is a constant
2. $\int_a^b f'(x)\,dx = f(b)-f(a)$

Upgrade for part I, applying the Chain Rule

If $F(x)=\int_a^{g(x)} f(t)\,dt$, then $\frac{dF}{dx}=\frac{d}{dx}\int_a^{g(x)} f(t)\,dt = f(g(x))g'(x)$.

For example, $\frac{d}{dx}\int_2^{x^3}\sin(t^2)\,dt = \left(\left(\sin(x^3)^2\right)\right)\left(3x^2\right)=3x^2\sin\left(x^6\right)$

An important alternate form for part II

$F(b)=F(a)+\int_a^b f(t)\,dt$

[Think of this as: ending value = starting value plus accumulation.]

For example, given that $\int_3^{12} f'(x)\,dx=-4$ and $f(3)=35$, find $f(12)$.

Using the alternate format, $f(12)=f(3)+\int_3^{12} f'(x)\,dx = 35+(-4)=31$.

Sample Problems

Multiple Choice – No Calculator

1. $\dfrac{d}{dx}\int_{2}^{x} \ln t \, dt =$

(A) $\ln x$ (B) $\ln 2$ (C) $\dfrac{1}{x}$

(D) $\dfrac{1}{2}$ (E) $\ln x - \ln 2$

2. If $g(x) = \int_{\pi}^{\pi x} \cos(t^2)\, dt$, then $g'(x) =$

(A) $\sin\left(\pi^2 x^2\right)$ (B) $\pi x \sin\left(\pi^2 x^2\right)$ (C) $\pi x \cos\left(\pi^2 x^2\right)$

(D) $\cos\left(\pi^2 x^2\right)$ (E) $\pi \cos\left(\pi^2 x^2\right)$

3. $\dfrac{d}{dx}\int_{\sin x}^{4} \sqrt{1+t^2}\, dt =$

(A) $\sqrt{1+\sin^2 x}$ (B) $-\cos x\sqrt{1+\sin^2 x}$ (C) $-\sqrt{1+\sin^2 x}$

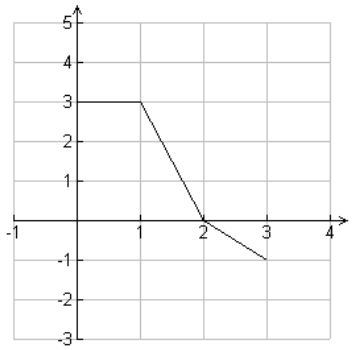
(D) $\cos x\sqrt{1+\sin^2 x}$ (E) $\sqrt{1+\cos^2 x}$

4. If f has two continuous derivatives on [5, 10], then $\int_{5}^{10} f''(t)\, dt =$

(A) $f'''(10) - f'''(5)$ (B) $f(10) - f(5)$ (C) $f'(10) - f'(5)$

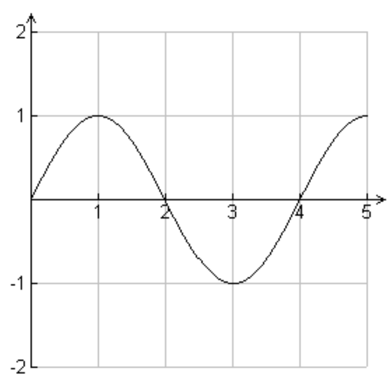
(D) $f''(10) - f''(5)$ (E) $f''(5) - f''(10)$

5. The graph of f is given, and g is an antiderivative of f. If $g(3)=6$, find $g(0)$.



(A) 1 (B) 2 (C) 4 (D) 5 (E) 10

6. The graph of f is given. $F(x)=\int_0^x f(t)\,dt$



Which of the following statements is true?
(A) F decreases on (1, 2).
(B) F has a relative minimum at $x = 2$
(C) F decreases on (2, 4)
(D) F has a relative maximum at $x = 1$.
(E) F has a point of inflection at $x = 4$.

7. $\dfrac{d}{dx}\displaystyle\int_{x}^{x^2} \tan(t)\, dt =$

(A) $\tan\left(x^2\right) - \tan x$ (B) $\tan x - \tan\left(x^2\right)$

(C) $\tan x - 2x\tan\left(x^2\right)$ (D) $2x\tan\left(x^2\right) - \tan x$

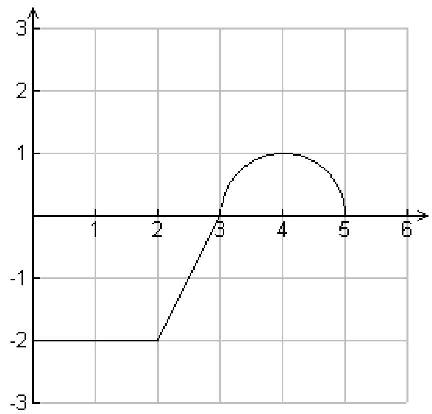
(E) $\sec^2\left(x^2\right) - \sec^2 x$

8. $\displaystyle\int_{1}^{e} \left(x - \frac{5}{x}\right) dx =$

(A) $\frac{1}{2}e^2 - \frac{11}{2}$ (B) $\frac{1}{2}e^2 - \frac{9}{2}$ (C) $e^2 - \frac{11}{2}$

(D) $\frac{1}{2}e^2 - \frac{3}{2}$ (E) $\frac{11}{2} - \frac{1}{2}e^2$

Free Response 1 – No Calculator



The graph of f is given. It consists of two line segments and a semi-circle.

$g(x) = \int_1^x f(t)\, dt$

(a) Find $g(0)$, $g(1)$, and $g(5)$.

(b) Find $g'(2)$, $g''(2)$, and $g'''(4)$ or state that it does not exist.

(c) For what value(s) of x does the graph of g have a point of inflection? Justify your answer.

(d) Find the absolute maximum and absolute minimum values of g on [0, 5]. Justify your answer.

Multiple Choice – Calculator Allowed

1. If $g(x)=\int_0^x \sin^2 t\, dt$, then $g'(2)=$

(A) 0 (B) 0.001 (C) 0 .173 (D) 0 .827 (E) 1.189

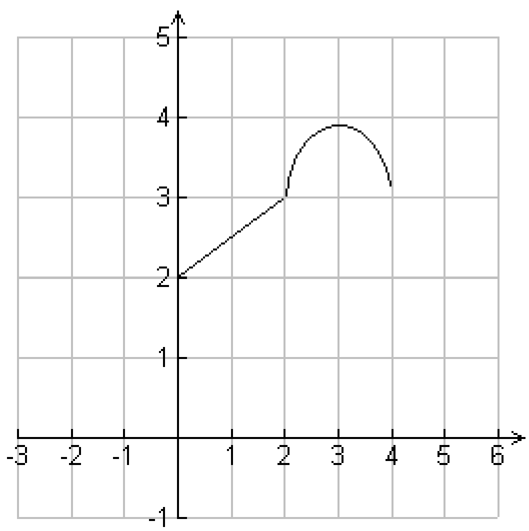
2. A car sold for \$16,000 and depreciated at a rate of $2e^{x^2}$ dollars per year. What is the value of the car 3 years after the purchase?

(A) \$206.17 (B) \$2889.09 (C) \$13,110.91

(D) \$16,206.17 (E) \$18,889.09

3. The graph of f is given, and *F(x)* is an antiderivative of *f*. If $\int_2^4 f(x)\, dx = 7.5$, find $F(4)-F(0)$.

(A) 1 (B) 1.5 (C) 7.5 (D) 12.5 (E) 18.5



4. The acceleration of an object in motion is defined by $\sqrt{1+t^2}$. The velocity at $t = 6$ is 22. Find the velocity at $t = 1$.

(A) 1.414 (B) 3.654 (C) 18.346 (D) 22.023 (E) 30.346

5. $h(x) = \int_1^x g(t)\,dt$ and $g(t) = \int_0^{t^2} \frac{\sqrt{1+u^2}}{u}\,du$. Find $h''(2.5)$.

(A) 1.013 (B) 1.077 (C) 2.154 (D) 5.064 (E) 12.659

6. Find $\int_{-2}^{2} f(x)\,dx$ if $f(x) = \begin{cases} 2x^2, & -2 \le x \le 0 \\ \sin 2x, & 0 < x \le 2 \end{cases}$

(A) 0 (B) 4.507 (C) 5.403 (D) 6.161 (E) 10.667

7. Let $g(x)$ be an antiderivative of $\frac{x^3}{\ln x}$. If $g(2) = 3$, find $g(6)$.

(A) 120.552 (B) 123.552 (C) 208.122

(D) 211.122 (E) 214.122

8. $h(x) = \int_0^{2x} (e^{\cos t} - 1)\,dt$ on (3, 6). On which interval(s) is h decreasing?

(A) (3.927, 5.498) (B) (5.498, 6)

(C) (3, 4.712) (D) Always decreasing on (3, 6)

(E) Never decreasing on (3, 6)

Free Response – Calculator Active

Let $g(x)=\int_{1}^{x}(5-8\sqrt{\ln t})\,dt$ for $x > 1$. Let $h(x)=\int_{1}^{x^2}(5-8\sqrt{\ln t})\,dt$ for $x > 1$.

(a) Write an equation of the tangent to g at $x = 3$.

(b) What is $h'(x)$?

(c) On which open interval(s) is g decreasing? Justify your answer?

(d) Find all x values for which h has relative extrema. Label them as maximum or minimum and justify your answer.

Key

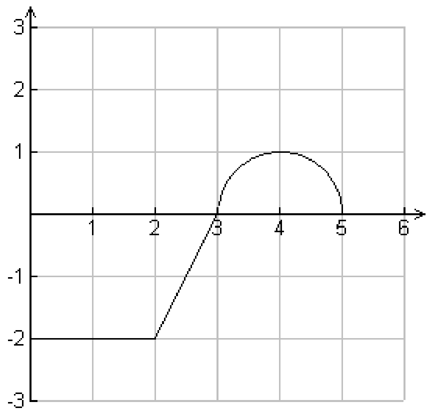
No Calculator

1. A
2. E
3. B
4. C
5. B
6. C
7. D
8. A

Calculator Allowed

1. D
2. C
3. D
4. B
5. D
6. D
7. E
8. A

Free Response 1 – No Calculator



The graph of f is given. It consists of two line segments and a semi-circle.

$g(x) = \int_1^x f(t)\,dt$

(a) Find $g(0)$, $g(1)$, and $g(5)$.

(b) Find $g'(2)$, $g''(2)$, and $g'''(4)$ or state that it does not exist.

(c) For what value(s) of x does the graph of g have a point of inflection? Justify your answer.

(d) Find the absolute maximum and absolute minimum values of g on [0, 5]. Justify your answer.

(a) $g(0) = \int_1^0 f(t)\,dt = 2$

$g(1) = \int_1^1 f(t)\,dt = 0$

$g(5) = \int_1^5 f(t)\,dt = \frac{1}{2}\pi - 3$

2 pts: 1 pt $g(0)$
1 pt $g(1)$ and $g(5)$

(b) $g'(2) = f(2) = -2$

$g''(2) = f'(2) = \text{DNE}$

$g''(4) = f'(4) = 0$

2 pts: 1 pt $g''(2)$
1 pt $g'(2)$ and $g''(4)$

(c) g has a point of inflection at $x = 4$ because $g' = f$ changes from increasing to decreasing.

2 pts: 1 pt $x = 4$
1 pt justification

(d) Candidates are $x = 0, 3, 5$, the endpoints of the interval and the critical number.

x	$g(x)$
0	2
3	-3
5	$\frac{1}{2}\pi - 3$

The absolute minimum value is −3.
The absolute maximum value is 2.

3 pts: 1 pt for candidates
1 pt evaluating candidates
1 pt for answers

Free Response 1 – Calculator Allowed

Let $g(x) = \int_1^x (5 - 8\sqrt{\ln t})\, dt$ for $x > 1$. Let $h(x) = \int_1^{x^2} (5 - 8\sqrt{\ln t})\, dt$ for $x > 1$.

(a) Write an equation of the tangent to g at $x = 3$.
(b) What is $h'(x)$?
(c) On which open interval(s) is g decreasing? Justify your answer?
(d) Find all x values for which h has relative extrema. Label them as maximum or minimum and justify your answer.

(a) $y + 2.354 = -3.385(x - 3)$	3 pts: 1 pt $g(3) = -2.354$ 1 pt $g'(3) = -3.385$ 1 pt equation
(b) $h'(x) = 2x\left(5 - 8\sqrt{\ln x^2}\right)$	2 pts for $h'(x)$
(c) g is decreasing where $g'(x) < 0$ $g'(x) = 5 - 8\sqrt{\ln t}$ $(1.4779, \infty)$	2 pts: 1 pt correct interval 1 pt justification
(d) h has a relative maximum at $x = 1.2156$ because h' changes sign from positive to negative.	2 pts: 1 pt correct relative maximum 1 pt justification

AP Calculus Exam Connections

The list below identifies free response questions that have been previously asked on the topic of the Fundamental Theorems of Calculus. These questions are available from the CollegeBoard and can be downloaded free of charge from AP Central.

http://apcentral.collegeboard.com.

Free Response Questions	
2004	AB Question 5
2005 Form B	AB Question 4
2006	AB Question 3