Pre-AP Calculus Summer Review Packet

Name _____

This is the description from the College Board for the prerequisites that students should have before taking AP Calculus.

> "Before studying calculus, all students should complete four years of secondary mathematics designed for college-bound students: courses in which they study algebra, geometry, trigonometry, analytic geometry, and elementary functions. These functions include linear, polynomial, rational, exponential, logarithmic, trigonometric, inverse trigonometric, and piecewise-defined functions. In particular, before studying calculus, students must be familiar with the properties of functions, the algebra of functions, and the graphs of functions. Students must also understand the language of functions (domain and range, odd and even, periodic, symmetry, zeros, intercepts, and so on) and know the values of the trigonometric functions at the numbers 0, $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$, $\frac{\pi}{2}$, and their multiples."

from http://media.collegeboard.com/digitalServices/pdf/ap/ap-calculus-course-description.pdf

This review packet is intended to review the topics listed above that you should be very familiar and proficient in at this point in your math career.

Please work out these problems and review these topics before the first day of AP Calculus class. I would recommend starting this packet a couple weeks before the summer is finished to refresh your brain of these mathematical concepts.

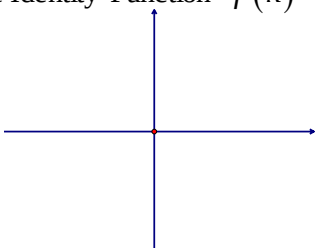
On the **first day of class** we will go over the syllabus and I will answer any questions you have on the review packet.

On the **second day of class** you will have a small 45 minute **test** over the topics on the review packet and we will begin Calculus topics post haste following the test.

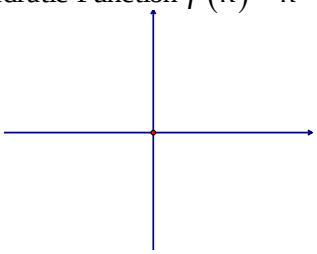
Graphs

You need to be very familiar with the following functions (domain, range, any asymptotes and what the graph looks like). Please **sketch** a graph of each function on the axis provided (label the axis).

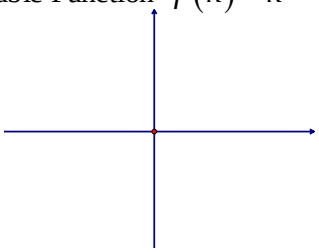
The Identity Function $f(x)=x$



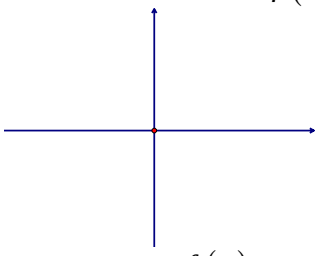
Quadratic Function $f(x)=x^2$



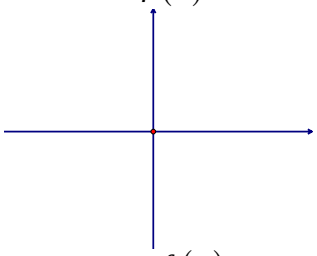
Cubic Function $f(x)=x^3$



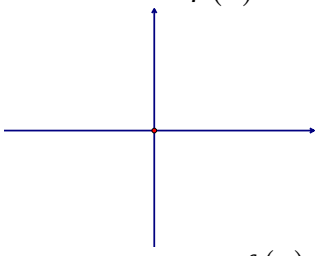
Absolute Value Function $f(x)=|x|$



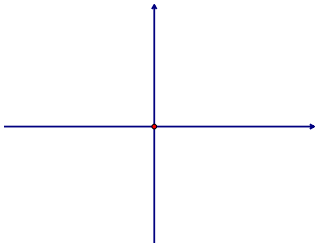
Sine Function $f(x)=\sin x$



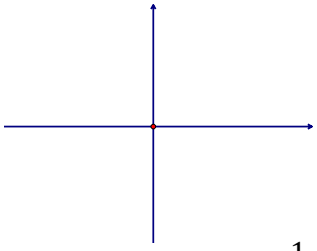
Cosine Function $f(x)=\cos x$



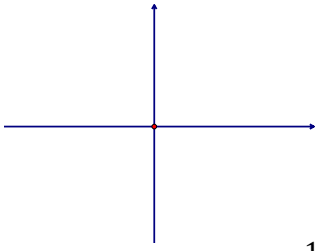
Tangent Function $f(x)=\tan x$



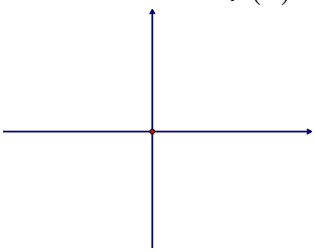
Secant Function $f(x)=\sec x$



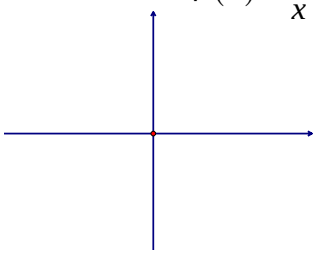
Exponential Function $f(x)=2^x$



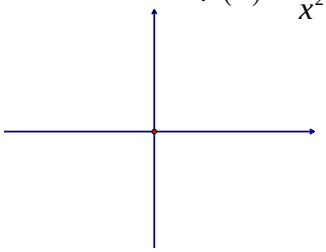
Logarithmic Function $f(x)=\log_2(x)$



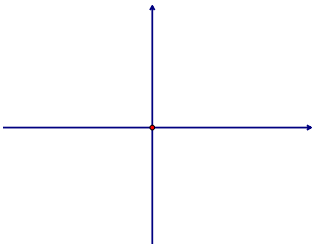
Rational Functions $f(x)=\dfrac{1}{x}$



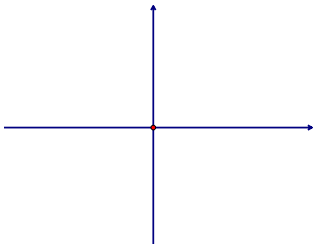
Rational Function $f(x)=\dfrac{1}{x^2}$



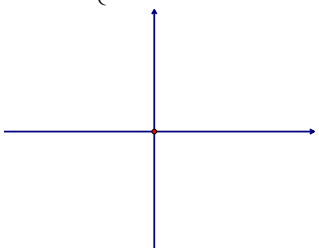
Square Root Function $f(x)=\sqrt{x}$



* $f(x)=\sqrt{a^2-x^2}$



* $f(x)=\begin{cases}0, & x \text{ is rational}\\ 1, & x \text{ is irrational}\end{cases}$



* these might be a little more difficult to sketch.

Understand the meaning of even and odd graphically and numerically.

Even: symmetry over the *y*-axis: $f(-x) = f(x)$

Odd: symmetry with respect to the origin: $f(-x) = -f(x)$

Fill in the following chart based on the graphs you sketched.

Function	Domain	Range $y = f(x)$	Symmetry with respect to the *y*-axis or origin	Even or Odd Function – $f(-x)=f(x)$ or $f(-x)=-f(x)$	Is the function periodic? If so, state the period.	Is $f(x)$ a one-to-one mapping? (for each $f(x)$ only one x exists) - horizontal line test -	State the x coordinates of any points of discontinuity.
$f(x) = x$							
$f(x) = x^2$							
$f(x) = x^3$							
$f(x) = \lvert x \rvert$							
$f(x) = \sin x$							
$f(x) = \cos x$							
$f(x) = \tan x$							
$f(x) = \sec x$							
$f(x) = 2^x$							
$f(x) = \log_2 x$							
$f(x) = \frac{1}{x}$							
$f(x) = \frac{1}{x^2}$							
$f(x) = \sqrt{x}$							
$f(x) = \sqrt{a^2 - x^2}$							
$f(x) = \begin{cases} 0, x \text{ is rational} \\ 1, x \text{ is irrational} \end{cases}$							

Factoring

Factor the following completely.

1. $3x^2 - 12$
2. $x^3 - 8$
3. $2x^3 - 8x^2 + 6x$
4. $x^2 + 2x + 2$
5. $x^4 - 16$
6. $x^2 - a^2$
7. $3x^2 - 11x + 6$
8. $x^3 + 3x^2 - 25x - 75$
9. $3x^3 - 12xc^4$

Factor the numerator and denominator and simplify. Identify any discontinuities.

1. $\dfrac{x^2 + 3x - 10}{x - 2}$
2. $\dfrac{x^2 - 4x + 4}{x^2 - 4}$
3. $\dfrac{x^3 + 8}{x^2 - 4x - 12}$
4. $\dfrac{x^2 - a^2}{x^4 - a^4}$
5. $\dfrac{x^2 - b^2}{x^3 - b^3}$
6. $\dfrac{x^3 - 14x^2 + 40x}{x^3 - 4x^2 - 100x + 400}$

Using Function Notation

Given that $f(x)=3x^2-4x$ and $g(x)=4\sin\left(\frac{\pi}{2}x\right)$, evaluate the following (expand and simplify any expressions).

1. $f(-2)=$
2. $f(1)=$
3. $f(2)=$
4. $f(x+2)=$
5. $f(x+\Delta x)=$
6. $g(3)=$
7. $g(x+4)=$
8. $g(x+h)=$

Piecewise Functions

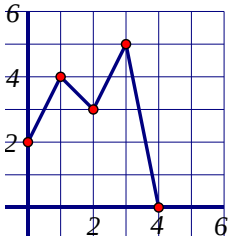
1. If $f(x)=\begin{cases} x^3-1, & x\le 1 \\ \cos(\pi x), & x>1 \end{cases}$, evaluate the following.
   a) $f(-1)=$
   b) $f(1)=$
   c) $f(3)=$
2. If $g(x)=\begin{cases} 3x+1, & x<2 \\ x^2-4, & x\ge 2 \end{cases}$, evaluate the following.
   a) $g(0)=$
   b) $g(2)=$
   c) $g(4)=$

3. If $h(x)=\begin{cases}\dfrac{x^2-4}{x-2}, & x\neq 2\\ 4, & x=2\end{cases}$, evaluate the following.

a) $h(2)=$

b) $h(3)=$

You need to be able to evaluate a function from a graph of a function. Here is some practice.



Graphed above is the function of $f(x)$.

Evaluate the following:

$f(1)=$ $h(-2)=$

$f(3)=$ $h(1)=$

Let $g(x)=f(x)+3x$. $g(1)=$ $p(0)=$

Let $h(x)=f(x+2)$. $g(4)=$ $p(1)=$

Let $p(x)=3x\cdot f(x)$. $p(3)=$

Linear Equations

Please write all equations in **Point-Slope Form** $y-y_1=m(x-x_1)$

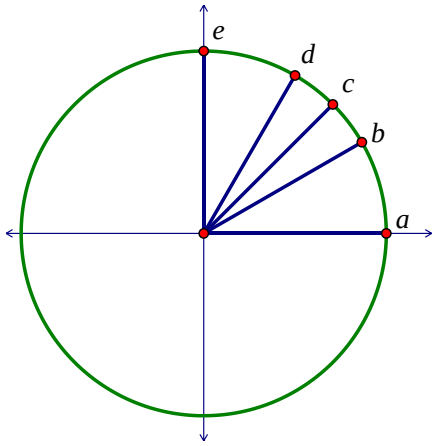
1. Find the equation of a line that passes through the point (8, 1) with slope $-\frac{3}{2}$.

2. Find an equation for the line passing through the points (3, 2) and (6, 7).

3. Find an equation for the line passing through the point (4,3) and perpendicular to the line $2x-3y=12$.

Trigonometry

The Unit Circle



	Degree	***Radian***	***Point*** (x, y)
a			
b			
c			
d			
e			

You should be able to determine these without a calculator. Find the exact value.

$\sin(0) =$ $\cos\left(\frac{\pi}{6}\right) =$ $\sec\left(\frac{\pi}{3}\right) =$ $\tan\left(\frac{\pi}{4}\right) =$

$\cos(0) =$ $\tan\left(-\frac{\pi}{3}\right) =$ $\csc\left(\frac{\pi}{4}\right) =$ $\cot\left(\frac{\pi}{2}\right) =$

$\sin\left(\frac{\pi}{2}\right) =$ $\sin(\pi) =$ $\tan(0) =$ $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) =$

$\cos\left(\frac{\pi}{2}\right) =$ $\cos(\pi) =$ $\sin\left(\frac{3\pi}{4}\right) =$ $\sin^{-1}\left(-\frac{1}{2}\right) =$

Trigonometric Identities (you should know these cold)

$\tan x = \frac{\sin x}{\cos x}$ $\sec x = \frac{1}{\cos x}$ $\sin^2 x + \cos^2 x = 1$

$\cot x = \frac{\cos x}{\sin x}$ $\csc x = \frac{1}{\sin x}$

Exponential and Logarithmic Functions

Laws of Exponents ($b > 0$)

	Rule	Complete the Equation and Simplify
Exponent zero	$b^0 = 1$	$5^0 =$
Products	$b^x b^y = b^{x+y}$	$3^2 \cdot 3^4 =$
Quotients	$\frac{b^x}{b^y} = b^{x-y}$	$\frac{2^{14}}{2^9} =$
Negative Exponents	$b^{-x} = \frac{1}{b^x}$	$4^{-3} =$
Power to a Power	$\left(b^x\right)^y = b^{xy}$	$\left(2^3\right)^4 =$
Roots	$b^{1/n} = \sqrt[n]{b}$	$12^{1/2} =$

Remember logarithmic functions are inverses of exponential functions: $b^{\log_b x} = x$ and $\log_b\left(b^x\right) = x$

Laws of Logarithms ($b > 0$)

	Rule	Complete the Equation and Simplify
Log of 1	$\log_b(1) = 0$	$\log_3(1) =$
Log of b	$\log_b(b) = 1$	$\log_4(4) =$
Products	$\log_b(xy) = \log_b x + \log_b y$	$\log_3(3 \cdot 2) =$
Quotients	$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$	$\log_2\left(\frac{4}{5}\right) =$
Reciprocals	$\log_b\left(\frac{1}{x}\right) = -\log_b x$	$\log_5\left(\frac{1}{125}\right) =$
Powers (any n)	$\log_b\left(x^n\right) = n \log_b x$	$\ln\left(a^3\right) =$

For more examples/practice look at the exercises on page 50 of your calculus textbook: (1-25 eoo, 27-33)